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Equitability, Allocation and Game Theory

Phil Stockton, Accord Energy Solutions Ltd
Allan Wilson, Smith Rea Energy Ltd

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1 INTRODUCTION

If not actually written into allocation agreements, equitability is often assumed to be one of their governing principles. The belief being that, if the system is equitable, then it should be free from bias and all participants treated equally.

But what exactly does equitability mean in this context and can it mean different things according to different viewpoints? This question has been addressed in detail, and at a fundamental level, by industries outside of oil and gas, in developing fair methods to allocate costs, resources and products.

Examples include:

- Civil engineering projects (Tennessee Valley Authority)
- Aircraft construction (McDonnell Douglas)
- Tree log allocation in the pulp and paper industry (several Finnish companies)
- Airport landing fees (Birmingham airport).

Many of these methods have been informed by the science of game theory.

This paper describes the application of these methods to hydrocarbon allocation in an effort to gain a deeper understanding of what is meant by the concept of equitability. The paper compares these methods with the more familiar proportional based approaches and explores instances when these traditional approaches may not appear equitable.

Three particular aspects of hydrocarbon allocation, fuel gas, the effects of commingling and access to restricted capacity, are used to illustrate the various approaches. These comparisons use data from real systems to assess how “fair” each appears.

In Section 2, the concept of equitability is examined, in particular the proportionality principle. A simple compression fuel gas allocation example is used to explore aspects of fairness. Section 3 describes alternative approaches to allocation borne out of co-operative game theory and describes their application to cost allocation in other industries. In Section 4 the alternative approaches are applied to the simplified example. Section 5 applies the various allocation approaches to data obtained from real allocation systems. Finally Section 6 provides a summary and some conclusions.

2 CONCEPTS OF EQUITABILITY

2.1 Proportionality

Proportionality is a deeply rooted concept in many areas such as law and business customs as a means of distributive justice. Examples include:

- When a firm goes bankrupt, creditors are repaid in proportion to the amounts they are owed
- If heirs to an estate are willed more than it is worth, they would normally inherit the estate in proportion to their bequests
- In 1987, the industrialised countries signed an accord to reduce their emissions of ozone damaging chemicals in proportion to their current emissions

- Metered oil and gas produced from a commingled process is frequently allocated in proportion to the amounts estimated to have been produced by each participant in the process.

The concept of proportionality dates back to the fourth century BC and the Greek philosopher, Aristotle who considered it to be a universal principle of distributive justice [1]. The dominance of the principle in Western culture owes much to Aristotle.

For it to be workable however, the quantity to be allocated needs to be divisible (e.g. oil or gas production, as opposed to the election of a member of parliament which is indivisible) and each claimant's entitlement should be expressible in some common metric (e.g. estimated oil production). When these two conditions are met proportionality appears the most reasonable method of allocation and is so deeply rooted in our ideas fairness that it is difficult to imagine any other method.

To gain some perspective however, it is useful to examine other cultures in which proportionality is not so prominent. A case in point is the Talmudic form of contracts which is almost as old as Aristotle's Ethics. The Babylonian Talmud is the collection of Jewish religious and legal decisions set down during the first five centuries A.D. The following problem was posed in the Talmud nearly 2,000 years ago (known as the "Contested Garment" problem):

"Two people have a claim on a garment; one claims it all and the other claims half, what is an equitable division of the garment?"

According to Aristotle's equity principle this would be split in the ratio of the claims, i.e. ⅔ to the first and ⅓ to the second claimant. According to the Talmud however, ¾ goes to the first and ¼ to the second. The reasoning for this is that only one half of the garment is contested and hence is split equally, the other half is uncontested and given to the one who claims it. In fact each claimant suffers an equal loss (i.e. ¼ of the garment).

The logic of the division in the Talmud is consistent with its precepts of fairness just as the Aristotle solution is in accordance with its precepts. The concept of what is fair is different in the two approaches and the example illustrates that what appears equitable can vary dependent on the properties a fair system is deemed to have.

A simple allocation example is used in the next section to illustrate some of the potential problems with the proportionality principle.

2.2 Simple Example: Compression Fuel Usage

Consider fuel gas allocation associated with a compressor on an offshore platform. In such allocation systems, various methods for the allocation of fuel gas can be observed; these include:

- In proportion to oil throughput
- In proportion to gas throughput
- In proportion to BOE production
- In proportion to estimated fuel usage.

Though it is acknowledged that fuel gas consumption on a platform may be allocated using any of the above metrics, some appear more equitable or fair than others. For example, allocation in proportion to oil production, when often compression fuel usage is the dominant fuel consumer on a platform, does appear to unfairly benefit high GOR fields at the expense of low GOR ones. Sometimes it appears a metric is selected upon which to base fuel allocation just because it is convenient rather than actually equitable.

From the options in the above list the most equitable must surely be deemed to be in proportion to estimated usage as the others may bear little relation to the actual fuel consumed as a result of processing each field's hydrocarbons.

At first sight therefore, the fairest estimation method might appear to be in proportion to estimated usage. But will this approach necessarily always seem fair and are there alternative approaches which are fairer?

Some of these issues can be illustrated with a simple example. Consider two offshore fields, called Neumann and Fisher[1] (in preference to the more anonymous A and B, etc.) being produced on a platform. The gas associated with each field is compressed in a single stage and the compressor is the principal consumer of fuel gas on the platform.

Figure 1 is a plot of power consumption versus throughput for an idealised centrifugal compressor:

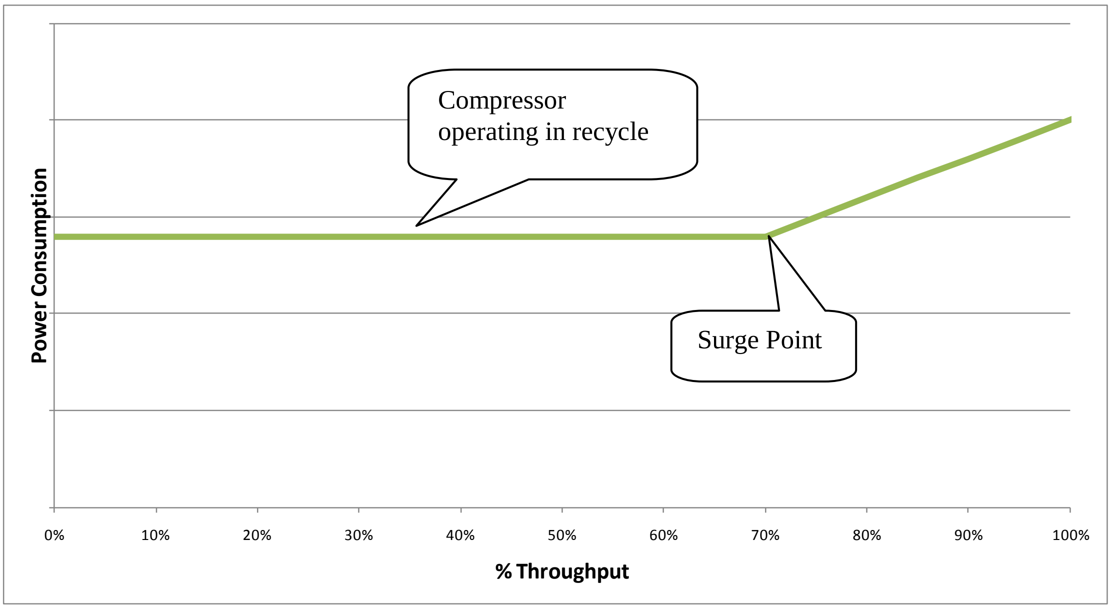

Figure 1 – Power Consumption as a Function of Compressor Throughput

Below approximately 70% of throughput the power consumption is constant. This is because the compressor has to operate in recycle mode below this point (surge point) and hence the actual compressor throughput is maintained at this minimum level.[2] This is illustrated schematically in Figure 2[3].

1 John von Neumann was one of the founding fathers of game theory. Ronald Fisher was credited with creating the foundations of modern statistics and also applied game theory to the study of animal behaviour.
2 Below this throughput the compressor experiences abnormal flows within its casing, loss of performance and possibly damage to the rotor blades.
3 In fact the flow rate dependent part of a real compressor power curve can be non-linear with power consumption actually falling as throughput increases towards the end of the curve. This is because the compressor efficiency normally improves with increasing flow towards some peak value.

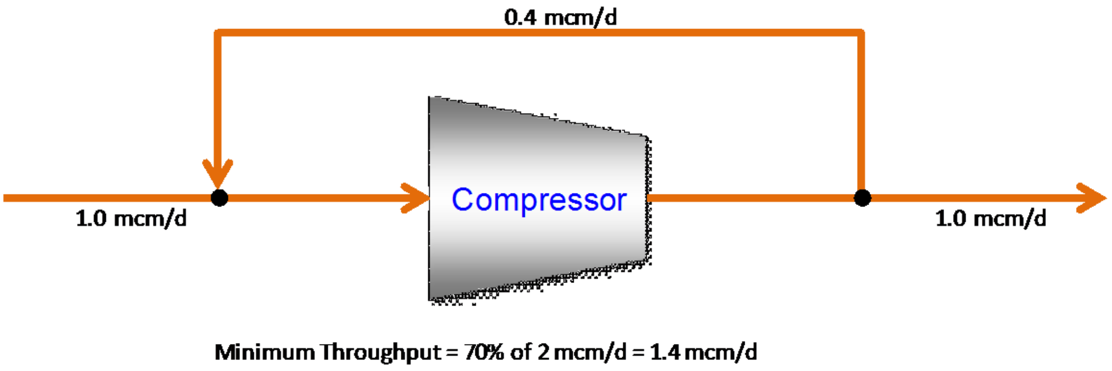

Figure 2 – Schematic of a Centrifugal Compressor Operating in Recycle Mode

The compressor capacity is deemed nominally equivalent to 2.0 mcm/d and hence the minimum flow is 70% of this, i.e. 1.4 mcm/d. In Figure 2 the gas production is only 1.0 mcm/d from the upstream process and hence the compressor has to recycle 0.4 mcm/d of gas. The fuel gas consumed by the compressor will be directly proportional to its power usage[4], so a plot of fuel consumed versus throughput has a similar shape to Figure 1. In this example, it is assumed that the compressor consumes 0.01 mcm/d of fuel for every 1 mcm/d of gas passing through it (i.e. 1% of throughput).

Say for example, Neumann is producing 0.8 mcm/d gas and Fisher 0.4 mcm/d. Since the production is only 1.2 mcm/d in total, the compressor would be recycling 0.2 mcm/d to maintain its minimum throughput of 1.4 mcm/d and hence consuming 0.014 mcm/d fuel gas. How should this be allocated between the two fields? Allocating in proportion to throughput, (0.00933 mcm/d to Neumann and 0.00467 mcm/d to Fisher), does not seem appropriate since the compressor fuel consumption at these throughputs is not dependent on field production. Indeed if either field was being produced alone it would incur an allocation of 0.014 mcm/d irrespective of its flowrate up to 1.4 mcm/d. Hence, since the fuel consumption is not flowrate dependent it seems more appropriate that fuel allocation should be equal, i.e. 0.007 mcm/d each.

Hence under this circumstance, the instinctive propensity to allocate proportionately does not seem as fair as allocating equally. This is because the chosen metric, gas production, does not have an impact on the fuel consumption at these rates and hence is an arbitrary basis.

However, consider what happens if Fisher production remains constant and Neumann increases. There will be no change in fuel consumption until Neumann's production exceeds 1.0 mcm/d, when the compression will no longer recycle and compressor fuel consumption will be directly proportional to total throughput. How should the fuel be allocated now? In proportion does not seem appropriate as the majority of the fuel consumption is due to maintenance of the minimum throughput. Perhaps 70% of the fuel should be divided equally and the remaining fuel divided pro rata to throughput. However, it could still be claimed by Neumann that if it was processed alone then the compressor would be in recycle and it is only the presence of Fisher that is rendering it beyond the surge point. Then there is the issue of what happens if Neumann's throughput increases above 1.6 mcm/d and a second compressor is started up in parallel. Both compressors would be in recycle and then should the second compressor's fuel be split?

Figure 3 plots the proportional allocation of fuel as a function of Neumann throughput:

4 Whether the compressor is driven directly by its own turbine or has an electric motor which is supplied power from an electrical generator, fuel gas will be consumed to power the compressor, at the generator or the turbine, and the consumption will be in proportion to the compressor throughput including any recycle.

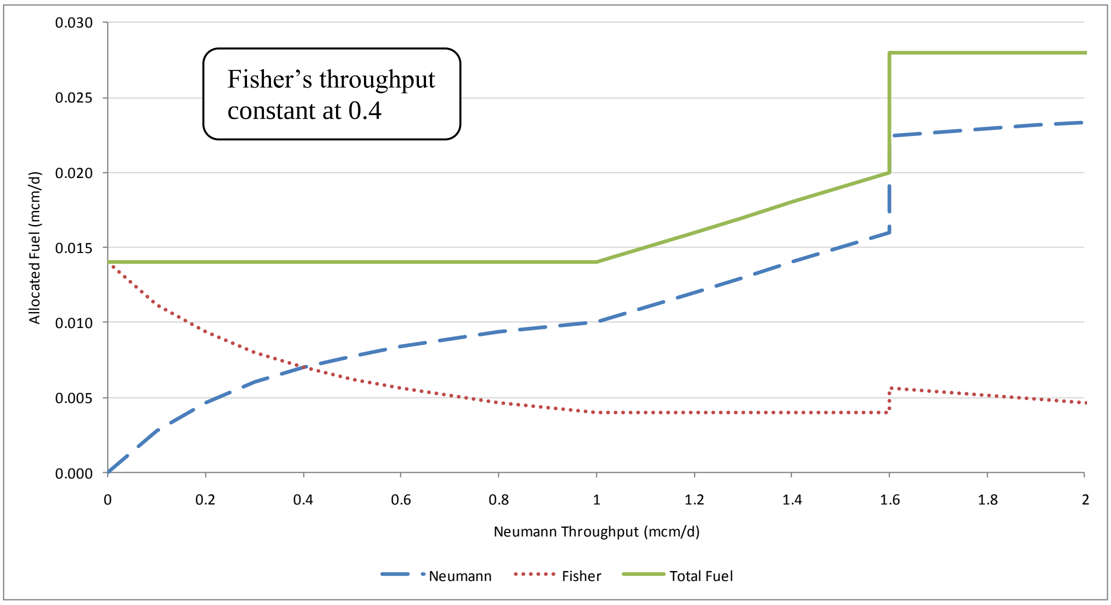

Figure 3 – Allocation of Fuel in Proportion to Field Throughput as a Function of Neumann Production

Two points appear incongruous from this plot:

- Despite the overall fuel usage remaining unchanged when the compressor is recycling Neumann's allocation increases and Fisher's falls.
- A rise in Neumann throughput above 1.6 mcm/d actually increases Fisher's fuel allocation due to the second compressor coming on stream.

Figure 4 is a reproduction of Figure 3, but also includes Neumann's fuel allocation if it alone had been compressed. Between 1.4 mcm/d and 1.6 mcm/d it is allocated the same fuel as when co-processed with Fisher. Between 1.6 mcm/d and 2.0 mcm/d it is allocated more fuel than it would have on a stand-alone basis. This is because in the comminlged case the second compressor is required. When the second compressor comes on stream, both fields may feel aggrieved at their fuel allocation for different reasons.

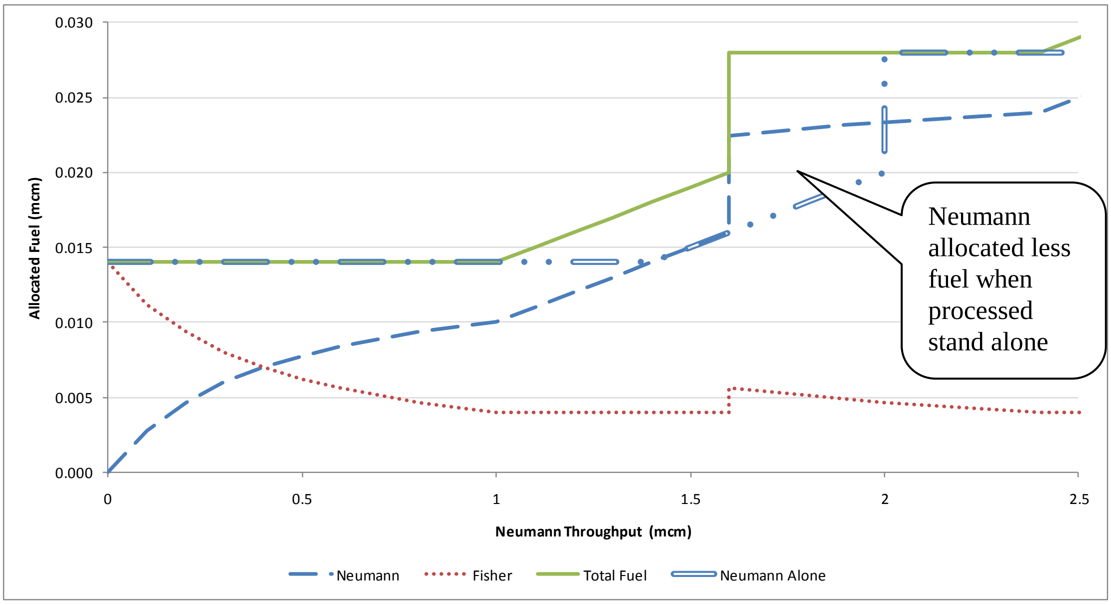

Figure 4 – Neumann Fuel Allocation Commingled versus Stand-Alone Production

The situation becomes more complicated when a third field, Nash[5], is brought on-line. Two scenarios serve to illustrate perceived problems with fairness. If the inclusion of Nash results in no increase in compressor fuel consumption, (i.e. the flowrates are such that the compressor(s) remains in recycle) then Neumann and Fisher will enjoy a reduction in fuel costs. Nash may claim that it should not pay any fuel costs as its introduction has not resulted in an increase in the consumption. This may appear unfair to Neumann and Fisher who might reasonably state that Nash should contribute to the costs.

Alternatively, the introduction of Nash may result in the start-up of another compressor and this may increase the allocation of fuel to Neumann and Fisher even though their operation remains unchanged. Under these circumstances, forcing Nash to pay its incremental costs may appeal to Neumann and Fisher. The problems posed by this example are not uncommon when a new field is tied back into an existing process. Addressing these increments in costs which seem to depend on the order in which fields join the process forms the basis of one of the alternative methods of allocation described later (in Section 3.4).

Two points may be concluded from the above analysis:

- Whatever fairness is in allocation it is not that easy to define and it can appear to depend on viewpoint.

- Allocating in proportion to throughput is not always equitable.

3 COST ALLOCATION AND CO-OPERATIVE GAME THEORY

Rather than assuming a method of allocation, co-operative game theory starts with the desirable properties an equitable method of allocation should have. It then mathematically derives methods based on these properties.

The following sections provide a high level discussion of the development of these methods and describe the properties they exhibit. More detailed analysis can be found in two books that address equitability and the application of game theory to cost allocation [6] and [7].

3.1 Game Theory

Game theory is the formal study of conflict and cooperation. Game theoretic concepts apply whenever the actions of several agents are interdependent. These agents may be individuals, groups, firms, or any combination of these. The concepts of game theory provide a language to formulate, structure, analyze, and understand strategic scenarios.

Though originally applied in the field of economics, game theory has been applied in sociology, as well as in biology (particularly evolutionary biology and ecology), engineering, political science, international relations, computer science, and philosophy. Game theory attempts to mathematically capture behaviour in strategic situations, or games, in which an individual's success in making choices depends on the choices of others.

Many of the concepts of game theory were developed by John Von Neumann and Oskar Morgernstern in their 1944 treatise “The Theory of Games and Economic Behaviour” [3].

The following sections describe some important concepts from game theory that have found applications in cost allocation in various industries. One of those concepts is the idea of the “The Core”. To illustrate the Core for three “players” in the game, it is convenient to introduce the triangular plot at this point.

5 John Nash developed many of the ideas of game theory and received a Nobel prize in 1994 for their application in the field of economic science.

3.2 Triangular Plot

The triangular plot (Figure 5) can be used to illustrate allocation among three fields.

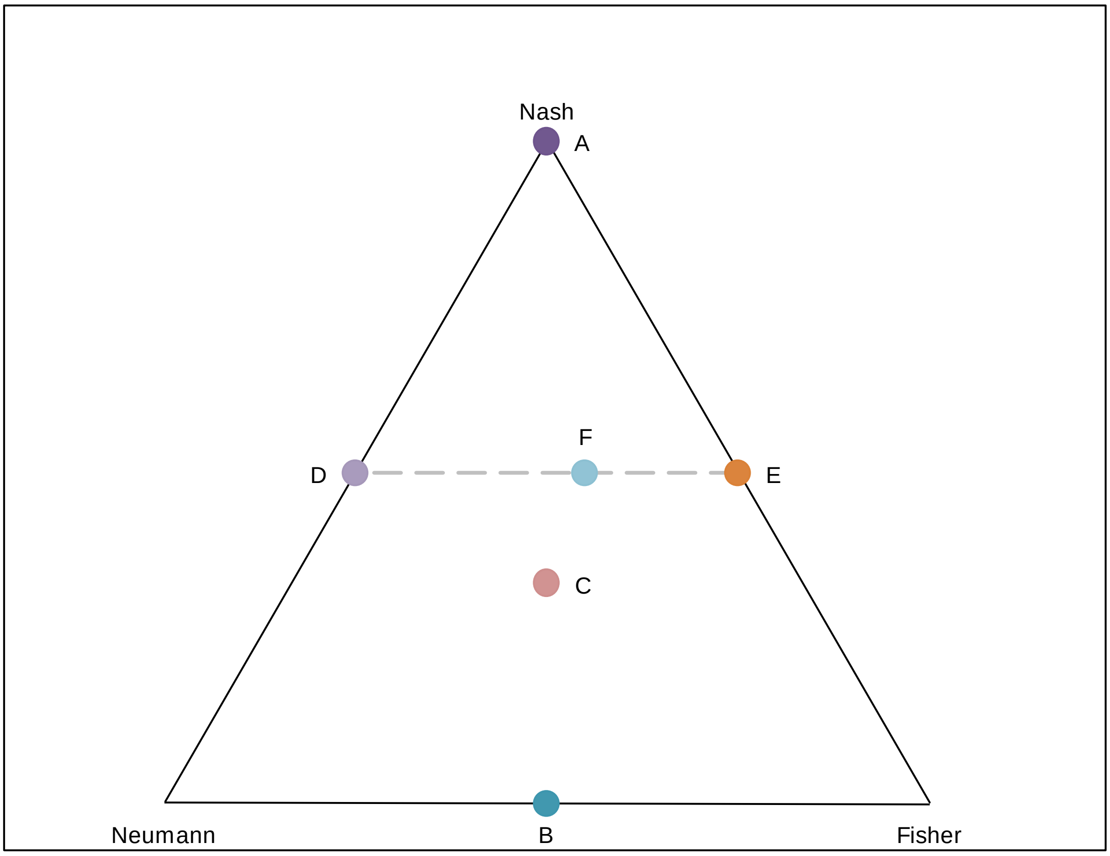

Figure 5 – Triangular Plot Illustration: Allocation to Neumann, Fisher and Nash

Each vertex of the triangle represents 100% of the fuel gas being allocated to one of the fields (as indicated by Point A where 100% is allocated to Nash). A point on the sides of the triangle represents allocation between two fields (as indicated by point B where half is allocated each to Neumann and Fisher and zero to Nash). Any point falling within the triangle represents allocation to all three fields as indicated by Point C where a third is allocated to all three fields. The closer the point is to a vertex the more is allocated to the associated field.

A line drawn parallel to the axis opposite the vertex represents a constant amount allocated to the associated field, for example any point on line D – E represents 50% being allocated to Nash. Points horizontally along the line represent how the remaining 50% is allocated to Neumann and Fisher. For example Point F represents 20% to Neumann, 30% to Fisher and 50% to Nash.

In summary, the closer a point is to the field's vertex, the more is allocated to that field.

3.3 The Core

In examining equitability it is important to identify properties an allocation method should have. Using the fuel gas allocation example, two such reasonable properties are:

1. Stand-Alone: No-one field shall be allocated more fuel than it would be if being processed alone.
2. Subsidy Free: The amount allocated to a field shall be greater than the incremental increase in fuel it causes when added to the other two fields being compressed. (If this is not true then the other two fields will be subsidizing the third field).

Satisfaction of these two conditions could reasonably be used to identify allocations as being fair and equitable. In essence, the economies of scale brought about by the sharing of facilities are enjoyed by all fields.

An allocation is said to be in the Core if it satisfies these two conditions and is illustrated on a triangular plot in Figure 6:

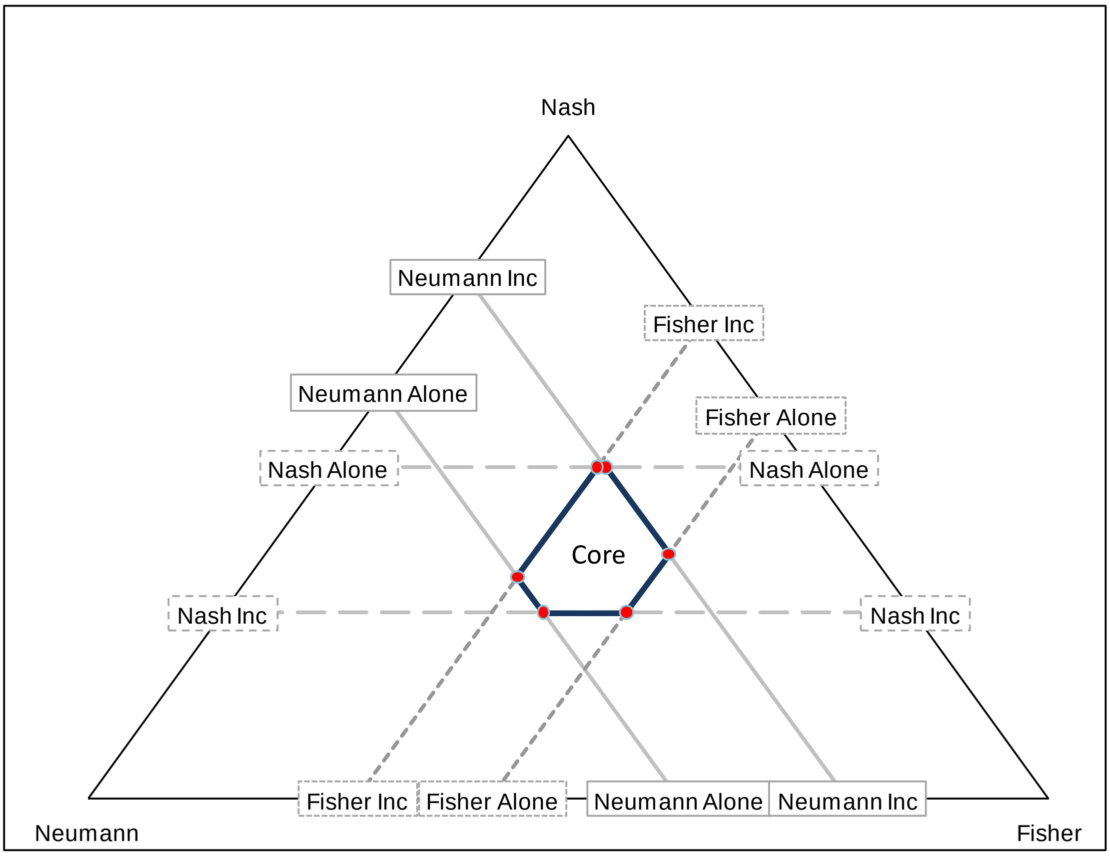

Figure 6 – Triangular Plot: The Core

The upper and lower fuel allocation lines are represented by the Alone (stand-alone) and Inc (incremental) lines respectively for each field. (The stand-alone fuel costs are greater and hence closer to the associated field's vertex than the incremental fuel costs). Points lying between these two limits for all three fields would satisfy the two conditions above. This bounded area (highlighted in the centre of the plot) is the Core.

Consider the simple example of Section 2.2, but now with three fields being processed. Field throughputs along with their stand-alone fuel costs and incremental fuel impacts are presented in Table 1.

Table 1 – Fuel Consumption and Allocation – Simplified 3 Field Example

		Neumann	Fisher	Nash	Total
Throughput	mcm/d	1.6	0.2	0.2	2.0
Stand-Alone Fuel	mcm/d	0.016	0.014	0.014	
Incremental Fuel	mcm/d	0.006	0.002	0.002	
Proportional Allocation	mcm/d	0.016	0.002	0.002	0.020

For the stand-alone fuel requirements, the compressor would be beyond the recycle point for Neumann whereas for Fisher and Nash the compressor would be recycling at 70% minimum flow.

The incremental fuel impact of 0.006 mcm/d for Neumann is calculated as the fuel consumed when all three are producing, i.e. 2.0 mcm/d throughput, compressor at full capacity consuming 0.020 mcm/d of fuel, less fuel consumption with just Fisher and Nash present, i.e. 0.4 mcm/d throughput, compressor recycling at 1.4 mcm/d and consuming 0.014 mcm/d fuel.

For both Fisher and Nash the incremental impact of 0.002 mcm/d is similarly calculated, i.e. 2.0 mcm/d, less the fuel consumption with just Neumann and either Fisher or Nash, which is at 1.8 mcm/d throughput, compressor consuming 0.018 mcm/d.

These upper and lower limits for each field are illustrated in Figure 7:

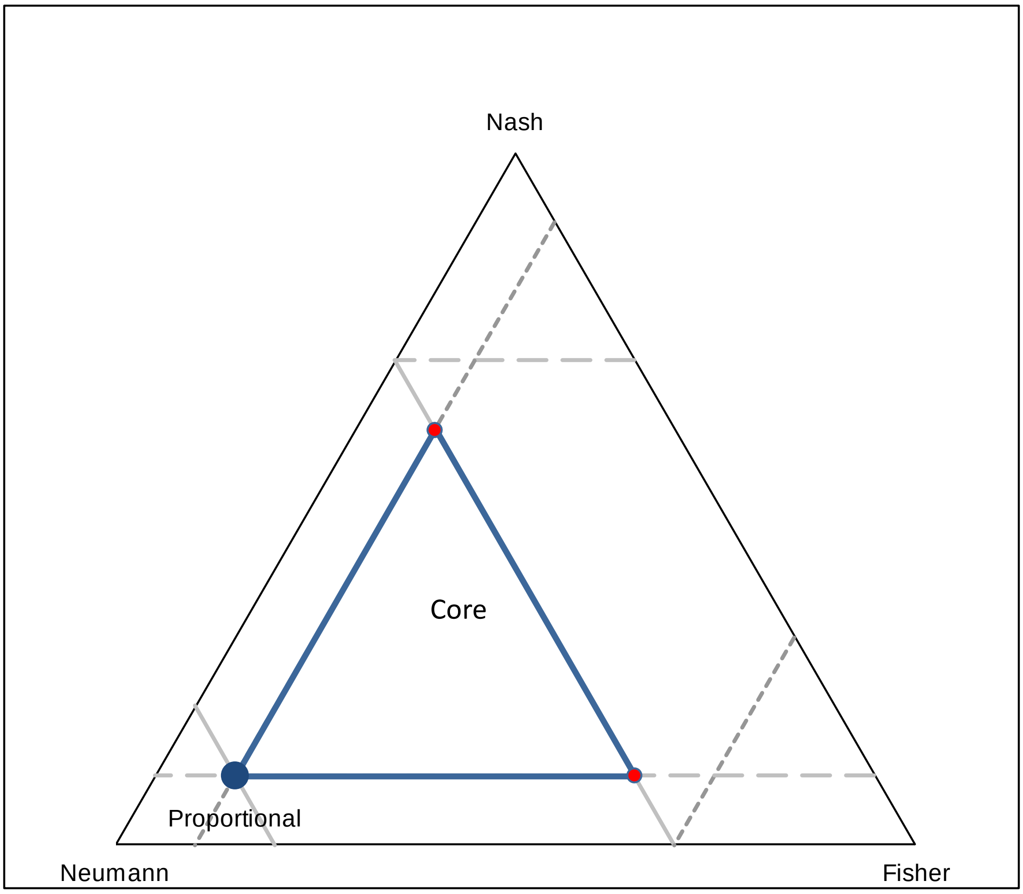

Figure 7 – Simplified 3 Field Example – Fuel Consumption Core and Proportional Allocation

As can be seen in this case the Core is delineated by Neumann's upper and lower limits and by Fisher and Nash's lower limits.

Also shown are the results of allocating the fuel in proportion to throughput located at the extreme vertex of the Core. The allocation is heavily weighted towards Neumann, i.e. favourable to both Fisher and Nash. The next two sections (3.4 and 3.5) discuss methods of allocation derived from co-operative game theory which result in allocations that appear more equitable in relation to the Core.

3.4 The Shapley value

As discussed in the example in Section 2.2 above, the order in which fields join the problem has a bearing on how the participants view the impact of the new entrant. Two extremes were discussed: the first was when the introduction of Nash caused no increase in fuel consumption compared with the case when it caused a disproportionate increase due to the start up of a second compressor.

The order in which fields join and the incremental impact they have on the fuel loading appears to have a bearing on how equitable the allocation is viewed. The Shapley value (devised by Lloyd Shapley in 1953 [4]) provides a methodology to account for order dependent incremental impacts.

The Shapley value may be expressed as the average marginal increase in fuel a field causes if each field joins the process one at a time.

Imagine the case where only Neumann is being processed; its marginal fuel cost would be its stand-alone fuel consumption. Then Fisher is introduced, the fuel may increase and this increase represents Fisher's marginal impact. Similarly when Nash joins, its marginal impact is calculated as any further increase in fuel consumption. The Shapley value is the average of each field's marginal impact over all possible orderings.

In order to calculate the Shapley value the estimated consumption for the cases where each combination of fields flowing has to be calculated and this is presented in Table 2 for the simple example described in Section 3.3:

Table 2 Fuel Gas Consumption for Combinations of Fields Flowing

Fields Flowing	Fuel Gas Consumprion mcm/d
Neumann, Fisher, Nash	0.020
Neumann, Fisher	0.018
Neumann, Nash	0.018
Fisher, Nash	0.014
Neumann	0.016
Fisher	0.014
Nash	0.014

The incremental impact on compressor fuel demand for each field in each of the 6 possible orderings is presented in Table 3:

Table 3 Incremental Compression Power Demand

Order of Processing					Incremental Fuel Consumption (mcm/d)		
1		2		3	Neumann	Fisher	Nash
Neumann	->	Fisher	->	Nash	0.016	0.002	0.002
Neumann	->	Nash	->	Fisher	0.016	0.002	0.002
Fisher	->	Neumann	->	Nash	0.004	0.014	0.002
Fisher	->	Nash	->	Neumann	0.006	0.014	0.000
Nash	->	Neumann	->	Fisher	0.004	0.002	0.014
Nash	->	Fisher	->	Neumann	0.006	0.000	0.014
				Average	0.0087	0.0057	0.0057
				% Share	43%	28%	28%

For example, in the top row, first Neumann is processed alone and hence consumes 0.0016 mcm/d of fuel. Fisher comes on stream and its impact is the difference between Neumann plus Fisher versus Neumann alone (0.018 mcm/d minus 0.016 mcm/d). Finally Nash comes on stream and its impact is the difference between all three being compressed (compressor at maximum throughput) and Neumann plus Fisher (0.020 mcm/d minus 0.018 mcm/d).

These incremental impacts are calculated for all 6 possible orderings of the fields and the average calculated: this is the Shapley value and is presented in Figure 8.

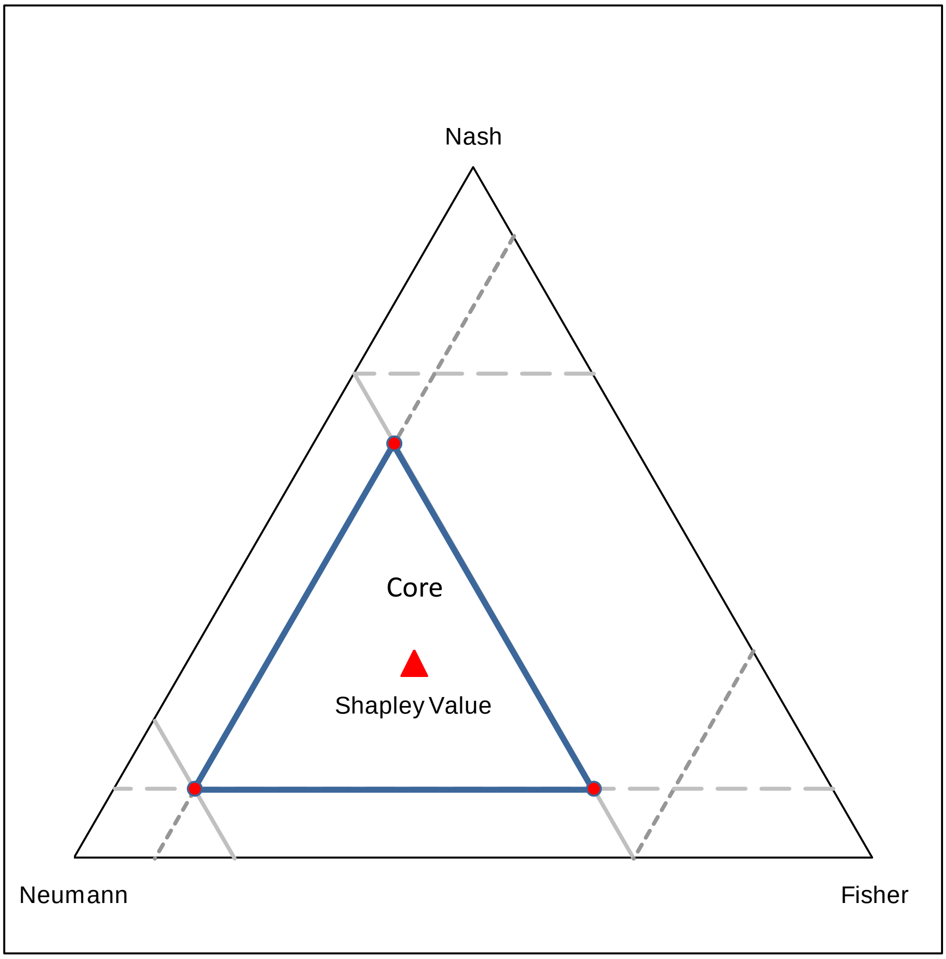

Figure 8 – Simplified 3 Field Example – Shapley value

The allocation according to the Shapley value is much more centrally located in the Core compared with that according to proportional allocation and weights the allocation more evenly between all three fields. The Shapley value reflects the fact that Fisher and Nash's individual and collective fuel consumption requires the compressor to be in recycle.

The Shapley value correctly reflects the average impact that each field has on fuel consumption over all possible orderings in which the fields are brought on stream.

A real life application of the Shapley value is the allocation of landing fees at Birmingham airport [9]. The landing fees are established to cover the costs of building and maintaining the runways. Equity demands that the landing fees reflect the burden that the different types of aircraft, using the airport, put on the system. Jumbo jets are assessed more than twin-engine Cessna's for example, because the larger planes require longer runways.

Though often in the Core, the Shapley value is not necessarily located within it. The next approach describes a method that uses the Core boundaries to provide an equitable allocation.

3.5 Equitable Core Solutions: The Nucleolus

It is not necessarily the case that there will always be a Core solution. When it does exist however, the next method is always located within it and shares the savings bounded by the Core equally among all fields.

The Core for the simplified 3 field example is reproduced below in Figure 9. Consider the case where the allocation to Nash is held constant at some fixed value represented by the horizontal dashed line indicated. The split of the remaining fuel gas between Neumann and Fisher is represented by any point along that line. A natural solution is to choose the midpoint of the line between the limits of the Core, and this represents an equal share of the savings.

Similarly Neumann's allocation could be held constant (indicated by the solid line) and Fisher and Nash split the remaining gas at the midpoint of the line bounded by the Core. Finally Fisher's held constant and Neumann and Nash split the savings.

If the quantities allocated to each field are adjusted so that the three lines intersect each other then the allocation is such that all three fields equally share the savings enjoyed as a result of co-processing. This point is represented by the green square in Figure 9 and is termed the nucleolus [5].

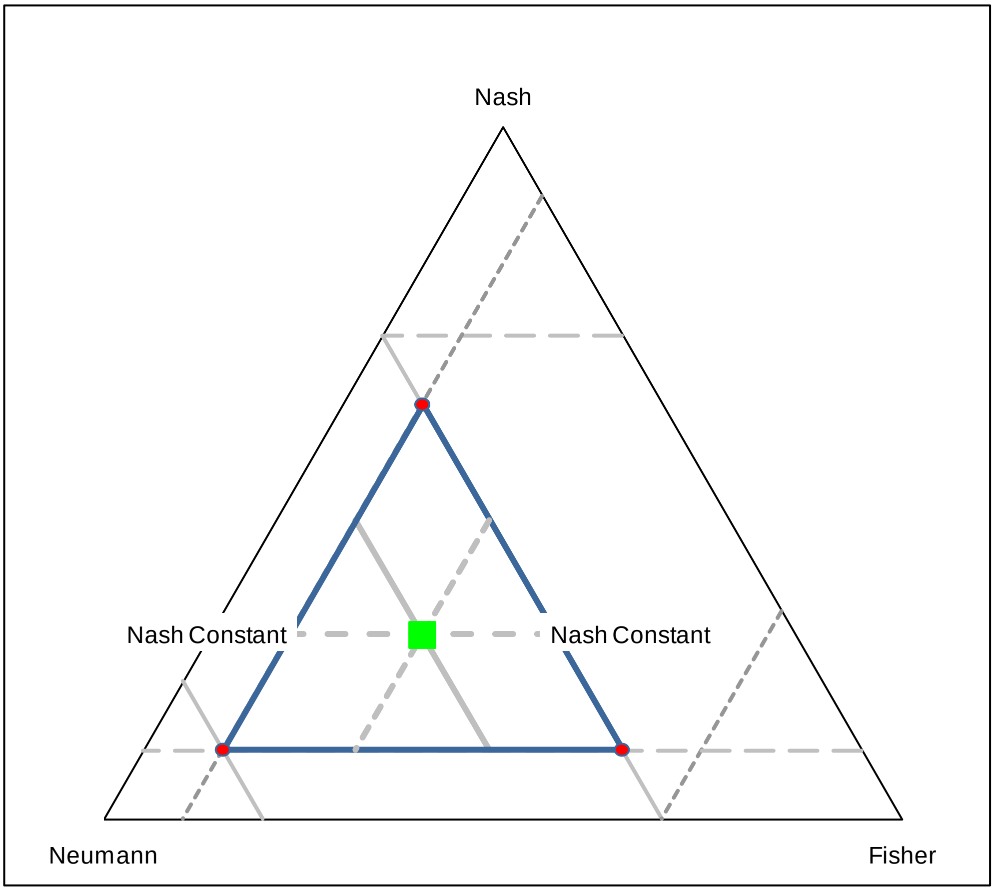

Figure 9 – Nucleolus – Simplified 3 Field Example

To calculate the nucleolus, the fuel consumption for each field and all combinations of fields being co-processed is required (this has already been presented in Table 2).

Now, consider any arbitrary allocation of fuel between the three fields. Each field should enjoy a saving in the allocation compared with the allocation it would have got if processed stand-alone. Similarly, any group of two fields' allocation added together should be less than the fuel consumed if both were processed without the third present.

Thus for all individual fields and groups of two, a comparison can be made between the sum of the allocated quantities and the fuel they would have incurred were only those fields being processed. Each such individual and group should enjoy some cost saving as a result of being in the allocation, otherwise it would be better off being processed alone or in combination with one other field. The nucleolus is the allocation that maximises the saving for the least well-off group. This is illustrated using the above example.

Imagine an allocation to the three fields is guessed at, say using the proportional values. The saving each individual field and each group incurs as a result of the allocation is calculated as the difference between these allocated values and the fuel loading for each combination of fields producing (i.e. each group). The results are presented in Table 4:

Table 4 Proportional Allocation: Field and Group Savings

Fields Flowing	Fuel Gas Consumprion	Nucleolus Allocation				
		Neumann	Fisher	Nash	Group	Saving
	mcm/d	mcm/d	mcm/d	mcm/d	mcm/d	mcm/d
		0.0160	**0.0020**	**0.0020**		0.0000
Neumann, Fisher, Nash	0.0200					
Neumann, Fisher	0.0180	0.0160	0.0020		0.0180	0.0000
Neumann, Nash	0.0180	0.0160		0.0020	0.0180	0.0000
Fisher, Nash	0.0140		0.0020	0.0020	0.0040	0.0100
Neumann	0.0160	0.0160			0.0160	0.0000
Fisher	0.0140		0.0020		0.0020	0.0120
Nash	0.0140			0.0020	0.0020	0.0120

For example, the sum of the Fisher Nash group's allocation is 0.004 mcm/d, but if they were processed on their own the fuel costs would be 0.014mcm/d. Hence, they enjoy a 0.01 mcm/d saving as a result of being in the allocation.

The minimum saving is zero and occurs in all groups containing Neumann. Can the minimum saving be improved by adjusting the allocation?

Since Nash and Fisher have the same throughputs their fuel allocation must be identical. Neumann's allocation must be the remainder of the total fuel consumed. Hence the allocation to Neumann can be varied and the minimum cost saving of all the groups calculated and plotted as a function of the Neumann throughput. This is presented in Figure 10.

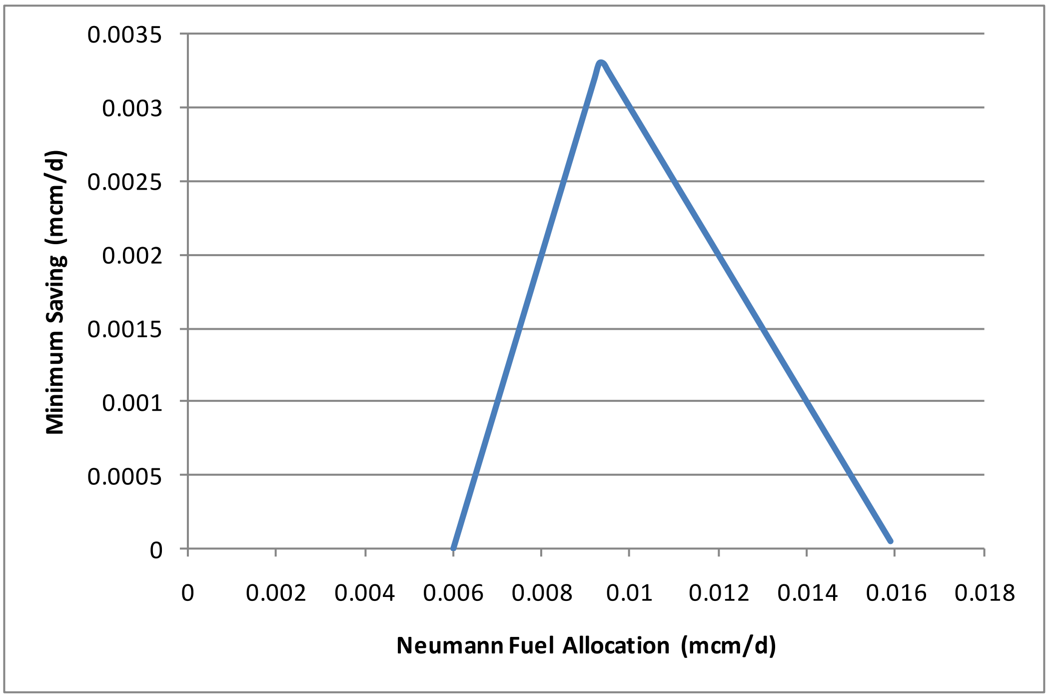

Figure 10 – Minimum Group Cost Saving – Simplified 3 Field Example

The optimal minimum saving occurs at 0.0093 mcm/d fuel allocated to Neumann and the allocation solution is presented in Table 5.

Table 5 Nucleolus Allocation: Field and Group Savings

Fields Flowing	Fuel Gas Consumprion	Nucleolus Allocation				
		Neumann	Fisher	Nash	Group	Saving
	mcm/d	mcm/d	mcm/d	mcm/d	mcm/d	mcm/d
		0.0093	**0.0053**	**0.0053**		0.0033
Neumann, Fisher, Nash	0.0200					
Neumann, Fisher	0.0180	0.0093	0.0053		0.0147	0.0033
Neumann, Nash	0.0180	0.0093		0.0053	0.0147	0.0033
Fisher, Nash	0.0140		0.0053	0.0053	0.0107	0.0033
Neumann	0.0160	0.0093			0.0093	0.0067
Fisher	0.0140		0.0053		0.0053	0.0087
Nash	0.0140			0.0053	0.0053	0.0087

All fields save fuel compared with their stand-alone fuel costs and all combinations of groups of two fields being co-processed experience a saving of 0.0033 mcm/d. Hence the savings are shared equally between all fields and possible groups they could form.

The solution to the nucleolus is a linear programming optimisation problem, where the minimum saving is maximised subject to the constraint that all the fuel is allocated. This is easily solved on a spreadsheet.

A case where the logic of the Core was applied to cost sharing was the Tennessee Valley Authority [10] in the 1930s. This was a US Government project to control flooding, provide hydroelectric power and improve navigation through a series of reservoirs in the Tennessee River basin. Economists charged with analysing the costs and benefits of the project were concerned with how to allocate common costs among the three objectives. It was stated that:

"The method should have a reasonable logical basis... It should not result in charging any objective with a greater investment than should suffice for its development at an alternate

single purpose site. Finally it should not charge any two or more objectives with a greater investment than would suffice for alternate dual or multiple purpose development".

In effect the second part of the statement describes the Core and in fact foreshadowed its formal game theoretic development.

The TVA asserted that the cost allocation was not based on any one mathematical formula, but on judgement. However, application of this "judgement" it was later realised had in fact allocated the costs using a variant of the nucleolus [11].

3.6 Comparison of Methods

The allocation results for the proportional, Shapley value and nucleolus are compared in Figure 11.

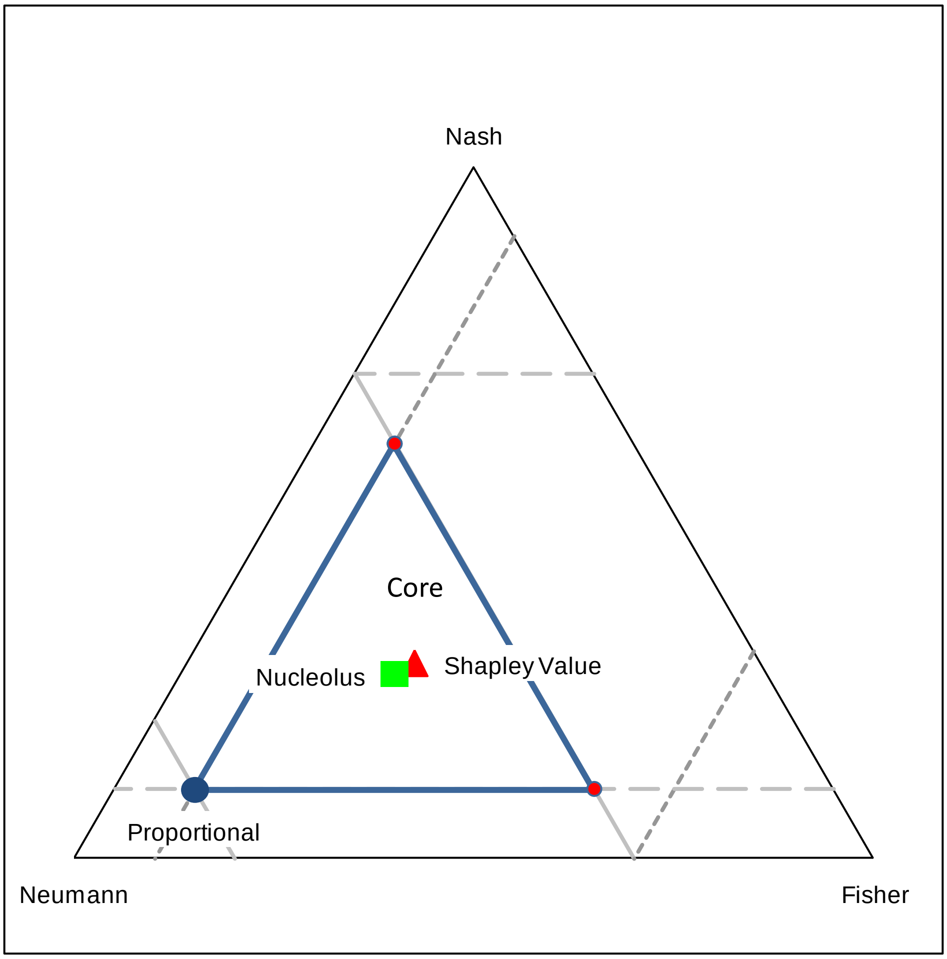

Figure 11 – Comparison of Compression Fuel Allocation – Simplified 3 Field Example

As can be seen the proportional approach heavily weights the allocation towards Neumann because of its relatively high throughput, despite the fact that much of the compressor fuel consumption is not flow dependent. As previously stated Neumann's fuel costs are as much as it would have incurred if processed alone and receives zero benefit as a result of sharing the compressor.

The Shapley value and nucleolus lie centrally in the core and provide similar though not exactly equal allocation solutions.

However, the question remains which one is the most equitable allocation? In essence there is no right or wrong answer and the correct one is whatever has been agreed. What can be said though is that one or other of these methods is the fairest if the allocation is deemed to have certain properties. These might include:

- The benefits of co-processing should be shared – this could be re-stated as the allocation should be in the Core
- The benefits of the co-processing should be shared equally
- The incremental impact of each field coming on stream should be accounted for

Each of the methods can satisfy some of these properties but none can be guaranteed to satisfy them all.

4 HYPOTHETICAL FUEL ALLOCATION EXAMPLES

Fuel gas allocation has been modelled for a variety of simple hypothetical production scenarios for Neumann, Fisher and Nash. In each scenario, fuel gas has been allocated proportionally to throughput and by calculating the Shapley value. The results of the model are intended to demonstrate the differences in allocation methods, and provide a simple means for understanding the features of allocation using the Shapley value, in particular fuel allocation dependency on throughput.

The allocation results using the nucleolus are omitted. Allocation according to the Shapley value and nucleolus give the same result in the two-field scenario. For three-field scenarios, allocation according to the Shapley value and nucleolus give similar, though not identical, results. The throughput dependent features of Shapley value allocation discussed here also apply to the nucleolus allocation method.

In the models gas compression is by a bank of identical compressors, each compressor is assumed to have operating parameters described in 2.2 and shown in Table 6.

Table 6 – Compressor Operating Parameters

Operating Parameter	Value
Compressor Capacity	2 mcm/d
Recycle Threshold	70%
Fuel usage	0.01 mcm/mcm of gas throughput

4.1 Fuel allocation to two producing fields

Fuel gas allocation has been modelled assuming production volumes of:

- Neumann: varying from 0 to 1.6 mcm/d;
- Fisher: constant at 0.4mcm/d.
- Nash: no production

Figure 12 shows compares Neumann's proportional and Shapley value allocation results as a function of Neumann throughput and Figure 13 shows the analogous results for Fisher.

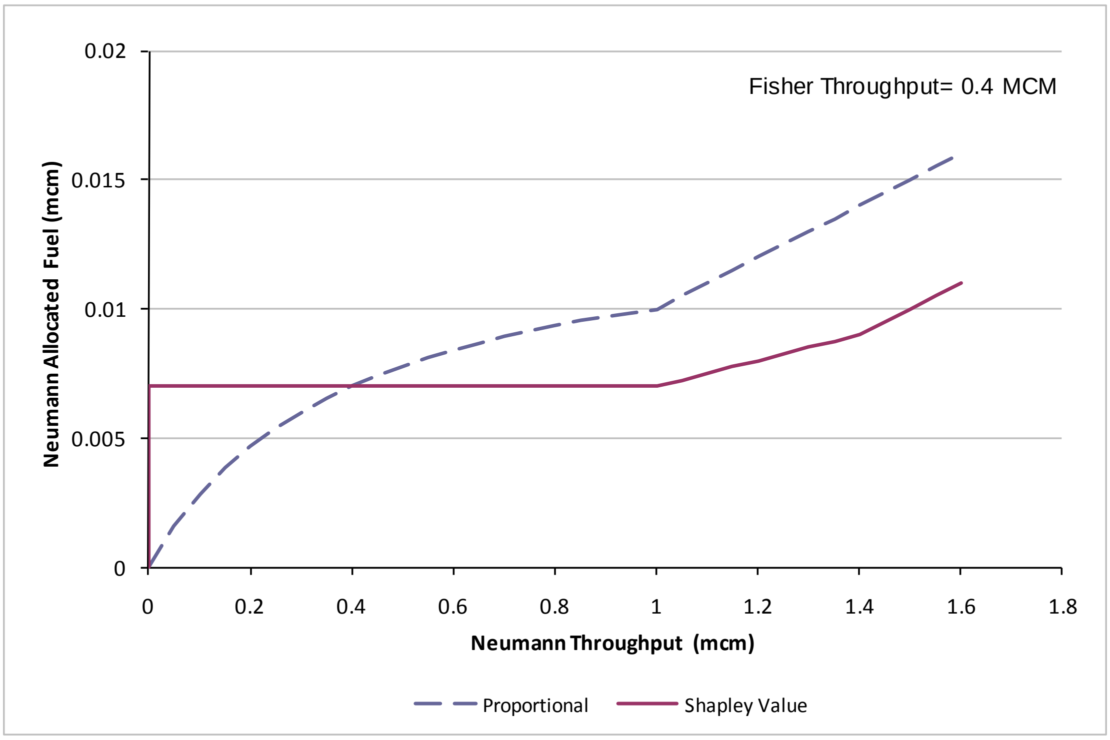

Figure 12 - Neumann fuel allocated in proportion to throughput and according to the Shapley value, plotted against Neumann throughput, when Fisher throughput is 0.4 mcm/d

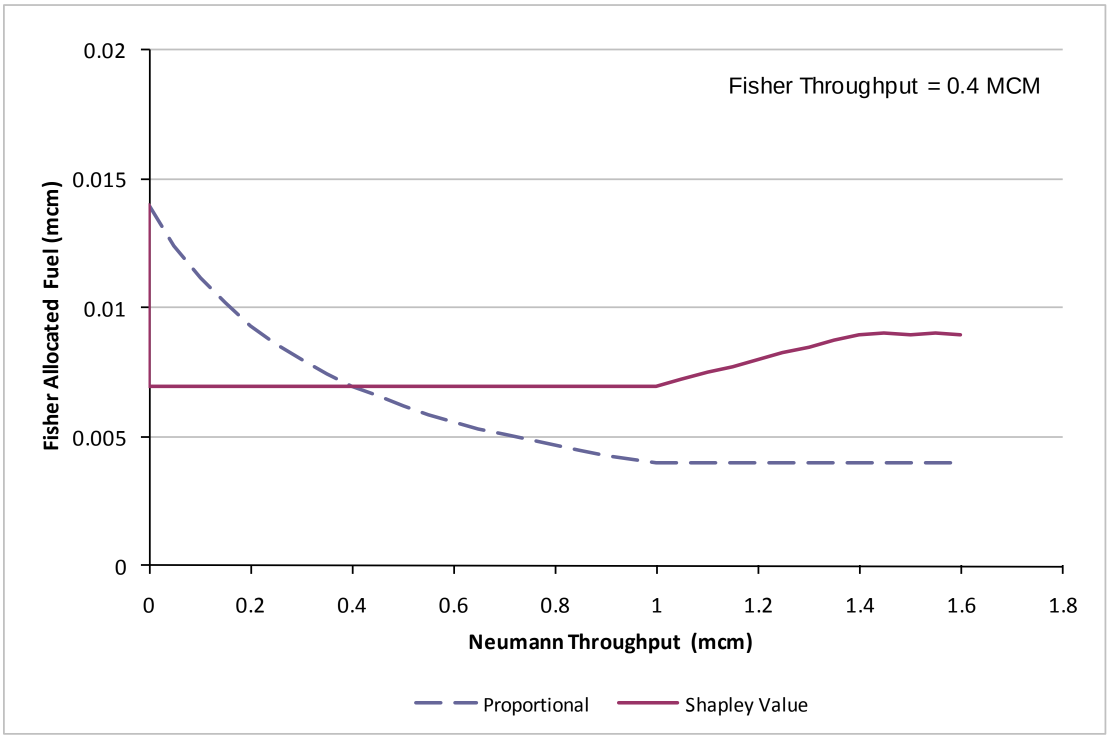

Figure 13 - Fisher fuel allocated in proportion to throughput and according to the Shapley value, plotted against Neumann throughput, when Fisher throughput is 0.4 mcm/d

In the proportional allocation case, Fisher fuel allocation decreases and Neumann fuel allocation increases as Neumann throughput increases until Neumann throughput reaches 1 mcm/d. Below that throughput the compressor is always on recycle so the total fuel usage to be allocated is constant. When the total Fisher and Neumann throughput crosses the compressor recycle threshold, fuel usage increases linearly with Neumann throughput while Fisher fuel allocation remains constant.

The dependency of Shapley value fuel allocation results on Neumann throughput are explained as follows:

$V_{Neumann}$ = 0 mcm/d. Fisher is allocated all fuel usage when Neumann is not producing.

0 < $V_{Neumann}$ < 1 mcm/d. As soon as Neumann starts producing, Fisher and Neumann are allocated half of the total fuel usage. Their fuel allocation remains at that level until $V_{Neumann}$ = 1 mcm/d at which point total throughput crosses the compressor recycle threshold. Shapley value allocation equates to the average of two fuel allocation results:-

- Fisher is producing at 0.4 mcm/d so its fuel usage is 0.014 mcm/d as the compressor must be on recycle. The marginal cost of Neumann then starting to produce is zero. Fisher fuel allocation is 0.014 mcm/d and Neumann fuel allocation is 0 mcm/d; and
- Neumann is producing in the range 0 to 1 mcm/d so its fuel usage is 0.014 mcm/d as the compressor must be on recycle. The marginal cost of Fisher then starting to produce is zero. Fisher fuel allocation is 0 mcm/d and Neumann fuel allocation is 0.014 mcm/d.

1 <= $V_{Neumann}$ < 1.4 mcm/d. The combined Fisher and Neumann throughput is now above the compressor recycle threshold. Both Fisher and Neumann fuel allocation are identical and increase with Neumann throughput. Shapley value allocation equates to the average of two fuel allocation results:-

- Fisher is producing at 0.4 mcm/d so its fuel usage is 0.014 mcm/d. The marginal cost of Neumann producing is ($V_{Neumann}$ + $V_{Fisherl}$)*0.01 – 0.014. Fisher fuel allocation is 0.014 mcm/d and Neumann fuel allocation is the marginal cost of Neumann production; and
- Neumann is producing in the range 0 to 1 mcm/d so its fuel usage is 0.014 mcm/d. The marginal cost of Fisher producing is ($V_{Neumann}$ + V_{Fisher})/ 100 – 0.014. Neumann fuel allocation is 0.014 mcm/d and Fisher fuel allocation is the marginal cost of Fisher production.

The Shapley value approach to allocation here is that although fuel usage is now higher, the increased fuel usage cannot be said to be due to Fisher or Neumann alone. It would seem unfair to pin the blame on one field for the increased fuel usage and in that sense it is fair, according to Shapley, to allocate fuel cost equally.

1.4 <= $V_{Neumann}$ < 1.6 mcm/d. Neumann throughput is now above the compressor recycle threshold. Increasing Neumann throughput does not however affect Fisher fuel allocation, which is constant for 1.4 < $V_{Neumann}$ < 1.6 mcm/d. Shapley value allocation equates to the average of two fuel allocation results:-

- Fisher is producing at 0.4 mcm/d so its fuel usage is 0.014 mcm/d. The marginal cost of Neumann producing is ($V_{Neumann}$ + $V_{Fisherl}$)*0.01 – 0.014. Fisher fuel allocation is 0.014 mcm/d and Neumann fuel allocation is the marginal cost of Neumann production; and
- Neumann is producing in the range 1.4 to 1.6 mcm/d so Neumann fuel allocation is 1% of Neumann throughput. The marginal cost of Fisher producing is 1% of Fisher throughput. Neumann fuel allocation is 1% of Neumann throughput and Fisher fuel allocation is the marginal cost of Fisher production.

According to the Shapley allocation method, as Neumann throughput is above the compressor recycle threshold, Neumann is allocated the fuel usage above the threshold.

Figure 12 and Figure 13 clearly demonstrate why it is worthwhile asking what constitutes a fair allocation methodology. Where fuel usage is throughput independent ($V_{Neumann}$ + V_{Fisher} <1.4 mcm/d) both Neumann and Fisher need the compressor to be on recycle consuming 0.014 mcm/d fuel, irrespective of the other field's production. In this instance it is arguably fairer to allocate fuel equally between Fisher and Neumann as Shapley does. If this is accepted then in comparison, it would seem that the use of proportional allocation results in Fisher subsidising Neumann fuel allocation for $V_{Neumann}$ < 0.4 mcm/d while Neumann subsidises Fisher's fuel allocation for $V_{Neumann}$ > 0.4 mcm/d.

As total throughput increases yet further and more stages of compression are required, the concepts of fairness and equity can be examined further. Figure 14 and Figure 15 show Neumann and Fisher fuel allocation as Neumann throughput is increased to 2.5 mcm/d while Fisher throughput remains at 0.4 mcm/d.

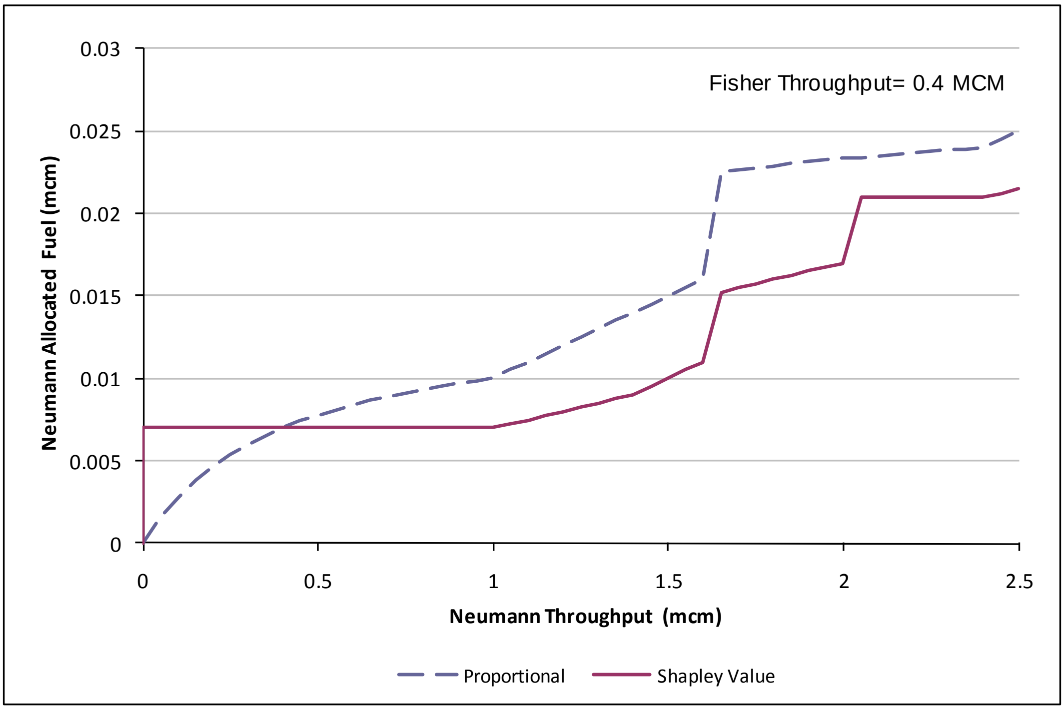

Figure 14 - Neumann fuel allocated in proportion to throughput and according to the Shapley value, plotted against Neumann throughput, when Fisher throughput is 0.4 mcm/d

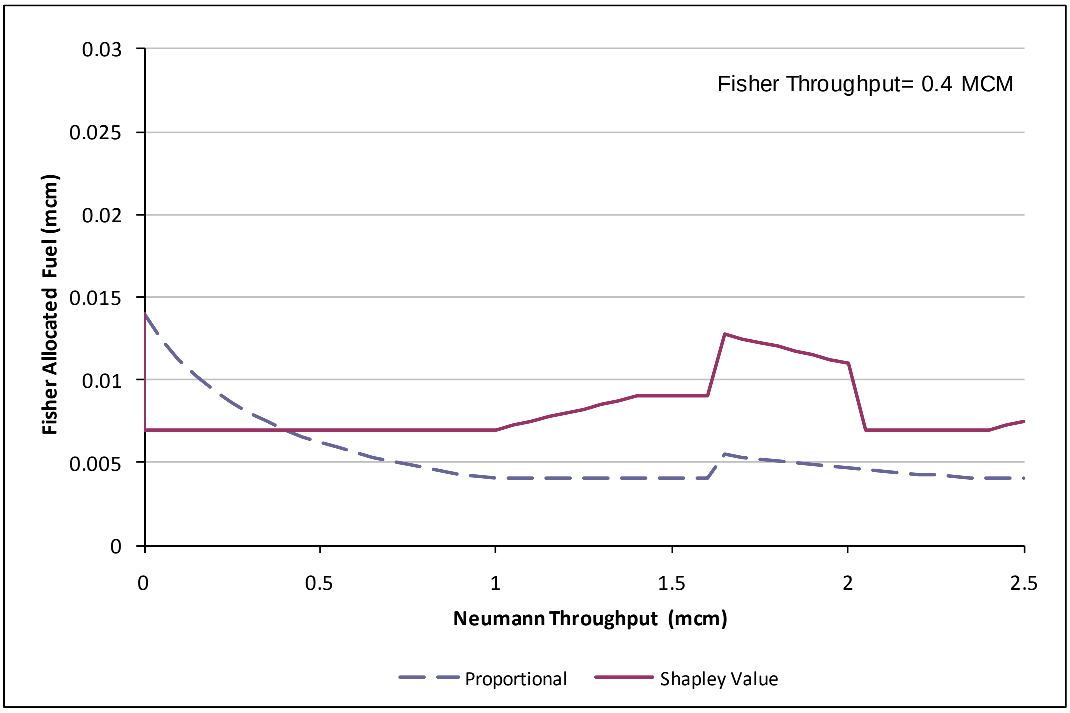

Figure 15 - Fisher fuel allocated in proportion to throughput and according to the Shapley value, plotted against Neumann throughput, when Fisher throughput is 0.4 mcm/d

For Neumann throughput above 1.6 mcm/d two compressors are now required both operating below their recycle threshold and the total fuel amount to be allocated is 0.028 mcm/d.

If fuel is allocated proportionally to throughput both Neumann's and Fisher's fuel allocation suddenly increase. As Neumann throughput increases further the proportion of fuel allocated to Neumann and Fisher increases and decreases respectively, until $V_{Neumann}$ = 2.4 mcm/d. At this point both compressors cross the recycle threshold and Fisher fuel allocation remains constant while Neumann fuel allocation increases with throughput.

The dependency of Shapley value fuel allocation results on Neumann throughput are explained as follows:

1.6 <= $V_{Neumann}$ < 2.0 mcm/d. Total throughput is now above a single compressor's capacity and both compressors are operating in recycle mode. Increasing Neumann throughput leads to higher Neuman and lower Fisher fuel allocation. Shapley value allocation equates to the average of two fuel allocation results:-

- Fisher is producing at 0.4 mcm/d so its fuel usage is 0.014 mcm/d. The marginal cost of Neumann producing is 0.014 mcm/d as the second compressor is required. Fisher and Neumann fuel allocation are each 0.014 mcm/d; and
- Neumann is producing in the range 1.6 to 2.0 mcm/d so its fuel usage is 1% of Neumann throughput. The marginal cost of Fisher producing is (0.028 – $V_{Neumann}$*0.01). Neumann fuel allocation is 1% of Neumann throughput and Fisher fuel allocation is the marginal cost of Fisher production.

2.0 <= $V_{Neumann}$ < 2.4 mcm/d. Neumann throughput is now above a single compressor's capacity though both compressors are still operating in recycle mode. Increasing Neumann throughput does not however affect Neumann or Fisher fuel allocation. Shapley value allocation equates to the average of two fuel allocation results:-

- Fisher is producing at 0.4 mcm/d so its fuel usage is 0.014 mcm/d. The marginal cost of Neumann producing is 0.014 mcm/d as the second compressor is required. Fisher and Neumann fuel allocation are each 0.014 mcm/d; and
- Neumann total fuel usage is 0.028 mcm/d. The marginal cost of Fisher throughput is zero as its inclusion does not require a further compressor online and both compressors remain operating in recycle mode even with Fisher online.

$V_{Neumann}$ >= 2.4 mcm/d. Total throughput is now above the compressor recycle threshold for both compressors. Increasing throughput leads to increased Neumann and Fisher fuel allocation. This situation is similar to 1 <= $V_{Neumann}$ < 1.4 mcm/d. No one field can be said to be entirely responsible for the increasing fuel costs so the increase in fuel allocation is shared equally between each field. Shapley value allocation equates to the average of two fuel allocation results:-

- Fisher is producing at 0.4 mcm/d so its fuel usage is 0.014 mcm/d. The marginal cost of Neumann producing is V_{Cap}*0.01 - 0.014 + ($V_{Neumann}$ + V_{Fisher} – V_{Cap})*0.01 = ($V_{Neumann}$+ V_{Fisher})*0.01 – 0.014. Fisher fuel allocation is 0.014 mcm/d and Neumann fuel allocation is the marginal cost of Neumann production; and
- Neumann requires two compressors on so its fuel usage is 0.028 mcm/d. The marginal cost of Fisher producing is ($V_{Neumann}$ + V_{Fisher})*0.01 – 0.028. Neumann fuel allocation is 0.028 mcm/d and Fisher fuel allocation is the marginal cost of Fisher production.

4.2 Fuel allocation to three producing fields

A similar analysis has been performed assuming Nash is also producing. In this scenario fuel gas allocation has been modelled assuming production volumes for Neumann, Fisher and Nash fields across the Neumann platform were:

- Neumann: varying from 0 to 1.6 mcm/d;

- Fisher: constant at 0.2 mcm/d;
- Nash: constant at 0.2 mcm/d.

Figure 16 shows the results of fuel gas allocation for Neumann plotted against Neumann throughput. Figure 17 shows the results of fuel gas allocation for Fisher plotted against Neumann throughput. In this scenario, Nash fuel allocation is identical to Fisher's and the Nash graphs have been omitted for brevity.

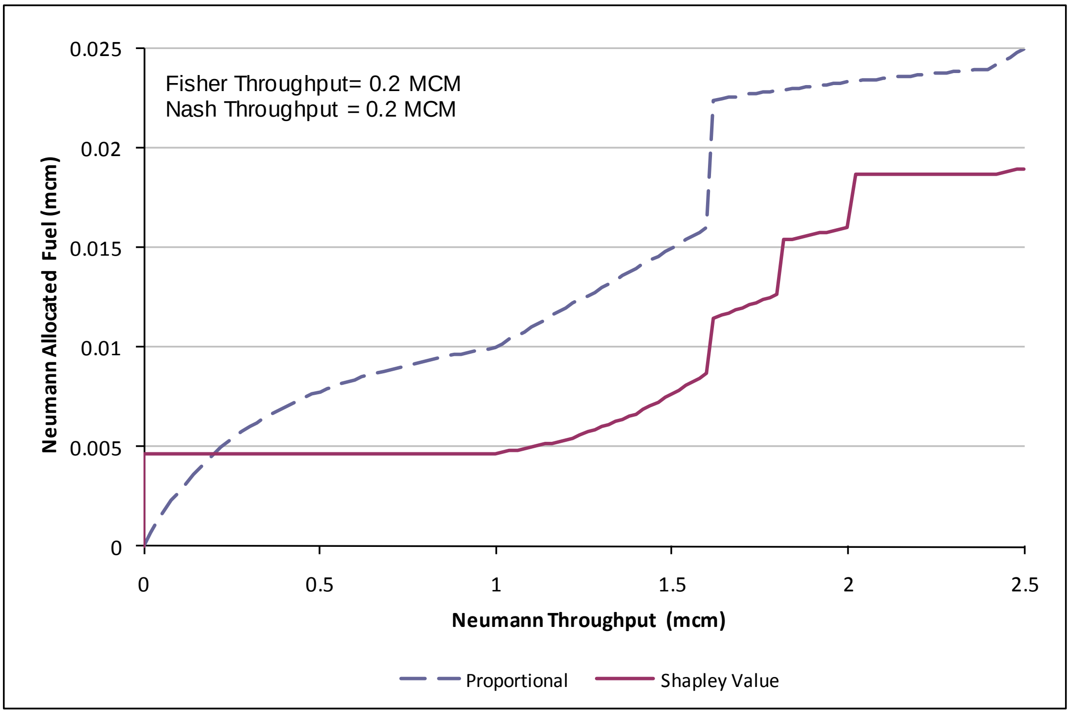

Figure 16 - Neumann fuel allocation plotted against Neumann throughput, when Fisher and Nash throughput are both 0.2 mcm/d

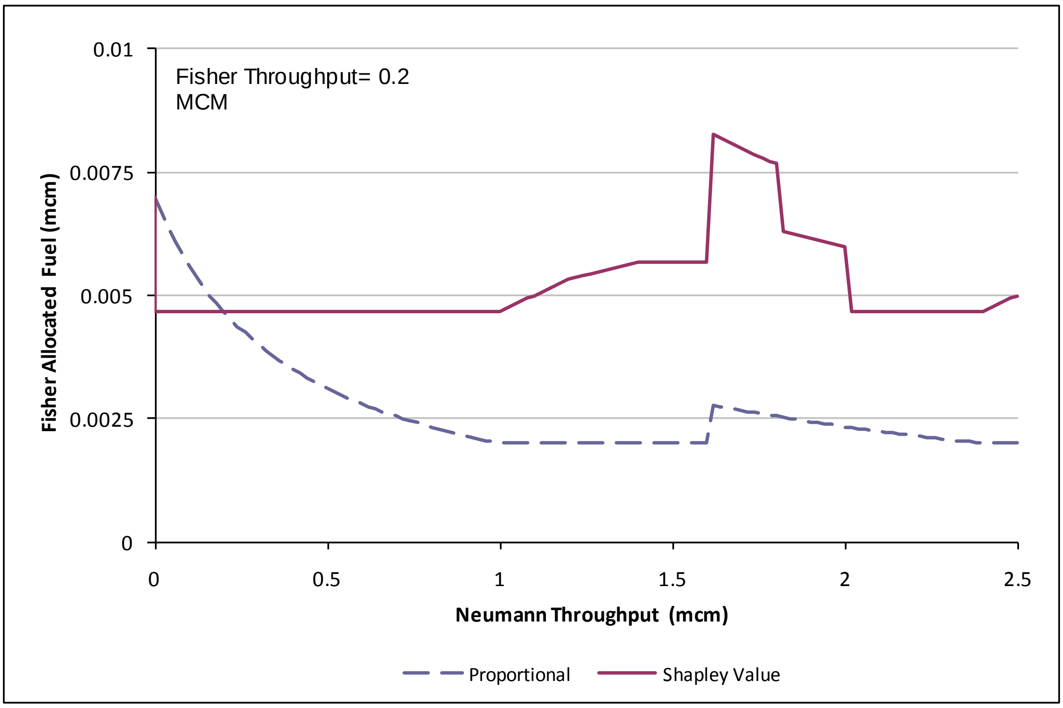

Figure 17 - Fisher fuel allocation plotted against Neumann throughput, when Fisher and Nash throughput are both 0.2 mcm/d

If fuel usage is allocated proportionately to throughput, similar features to the Neumann-Fisher two field scenario examined in Section 4.1 are observed. As would be expected summing the Fisher and Nash fuel allocation in this scenario gives the same results as presented in Section 4.1.

However, if fuel allocation is performed according to the Shapley method then the results of fuel allocation appear more complex than in the two field scenario. As Neumann throughput increases fuel allocation is subject to more step changes and throughput dependency. While Fisher and Nash are allocated fuel equally, it is not the case that summing the Fisher and Nash fuel allocation in this scenario gives the Fisher fuel allocation derived in Section 4.1.

For Neumann throughput up to 2.5 mcm/d, the fuel allocation results derived according to Shapley's method can be explained as follows:-

$V_{Neumann}$ = 0 mcm/d. Fuel usage is allocated equally to Fisher and Nash when Neumann is not producing.

0 < $V_{Neumann}$ < 1 mcm/d. As in the two field scenario the cost of Neumann's admission into the system is an equal share of total fuel usage with Fisher and Nash. Each field is allocated a third of total fuel usage. This remains true for Neumann throughputs up to the level where the total throughput crosses the compressor recycle threshold, in this case $V_{Neumann}$ = 1 mcm/d.

1 <= $V_{Neumann}$ < 1.2 mcm/d. The combined Neumann, Fisher and Nash throughput is now above the compressor recycle threshold. Increasing throughput leads to increased Neumann, Fisher and Nash fuel allocation. No one field can be said to be entirely responsible for the increasing fuel costs as throughput increases above the recycle threshold. Thus the increase in fuel allocation is shared equally between all three fields. Each field is still allocated a third of total fuel usage.

1.2 <= $V_{Neumann}$ < 1.4 mcm/d. The combined throughput of Neumann and any one of Fisher or Nash is now above the compressor recycle threshold. Increasing throughput leads to increased Neumann, Fisher and Nash fuel allocation. Notably, Neumann fuel allocation increases twice as fast as for 1 <= $V_{Neumman}$ < 1.2 mcm/d, and Fisher or Nash fuel allocation increases at half the rate. This is because Neumann now only needs production from one of Fisher and Nash for throughput to go above the recycle threshold. In contrast, the compressor is still below recycle threshold if the combination of Fisher and Nash production is considered together. It is only when Neumann production is added to throughput that the compressor recycle threshold is crossed.

1.4 <= $V_{Neumann}$ < 1.6 mcm/d. Neumann throughput is now above the compressor recycle threshold. Increasing Neumann throughput does not however affect Fisher or Nash fuel allocation, which is constant for 1.4 < $V_{Neumann}$ < 1.6 mcm/d. As in section 4.1, Neumann is allocated the fuel usage above the threshold.

1.6 <= $V_{Neumann}$ < 1.8 mcm/d. Total throughput is now above a single compressor's capacity so there is a corresponding step change in fuel allocation to each of Neumann, Fisher and Nash. However, since Fisher and Nash can produce together without exceeding the compressor capacity their fuel allocation is lower. The rate of change of Neumann fuel allocation with throughput is the same as for 1.2 < $V_{Neumann}$ < 1.4 mcm/d. As Neumann throughput increases Fisher and Nash are allocated less because Neumann's marginal cost increases.

1.8 <= $V_{Neumann}$ < 2.0 mcm/d. Throughput is now above a single compressor's capacity for either combination of Neumann and Fisher or Neumann and Nash and there is again a step change in the fuel allocation. Neumann is now allocated more fuel as it requires only one more field's production to cross the compressor capacity threshold. Fisher and Nash are allocated a correspondingly smaller amount. In addition the rate of change of fuel allocation for each of Neumann, Fisher and Nash is half of that in 1.6 < $V_{Neumann}$ < 1.8 mcm/d. As Neumann throughput increases Fisher and Nash are allocated less because Neumann' marginal cost increases.

2.0 <= $V_{Neumann}$ < 2.4 mcm/d. Neumann throughput is now above the single compressor capacity. There is again a step change in Neumann fuel allocation as it requires the second compressor to be online. Fisher and Nash are allocated correspondingly less. Fuel allocation remains constant as both stages of compression are in recycle mode.

$V_{Neumann}$ >= 2.4 mcm/d. The combined Neumann, Fisher and Nash throughput is now above the compressor recycle threshold on both compressors. Increasing throughput leads to increased Neumann, Fisher and Nash fuel allocation. This situation is similar to 1 <= $V_{Neumann}$ < 1.2 mcm/d. No one field can be said to be entirely responsible for the increasing fuel costs so the increase in fuel allocation is shared equally between all three fields

4.3 Comments on Fuel Allocation Scenarios

As stated before, it seems that proportional allocation is instinctively fair. But what do the modelled scenarios show us about fairness in fuel allocation?

If the assumptions upon which the Shapley allocation is based are seen as reasonable, then proportional allocation leads to fields subsidising each others fuel costs. As the models have shown, the subsidising field(s) and beneficiaries and the amount of subsidy varies with total throughput.

This can be seen, for example, at low throughput where fuel usage is independent of throughput and Shapley allocation results in all fields being allocated fuel equally. In the examples considered here, the subsidising field and beneficiary change as the total throughput crosses the compressor recycle threshold.

In scenarios where Fisher and Nash were modelled with higher throughputs (e.g. V_{Fisher} = V_{Nash} = 0.8 mcm and 1 mcm) then the amount of subsidy tends to become smaller than the examples in sections 4.1 and 4.2. The beneficiaries also alternated between fields as total throughput crossed compressor recycle thresholds and capacities.

The models have also illustrated the dependence of fuel allocation results with throughput when using Shapley allocation. The Shapley allocation results presented here exhibit step changes and linear dependence with throughput which are explained as being due to total throughput or a combination of field throughputs crossing compressor recycle thresholds and capacities.

5 OIL AND GAS ALLOCATION APPLICATIONS

The following examples use real data from actual allocation systems and compare the actual proportional based allocation results with those that would be obtained using the Shapley value and nucleolus.

The data has been anonymised using the field names Neumann, Fisher and Nash from the preceding simplified examples.

5.1 Fuel Gas Allocation for a Compression System

This example concerns an offshore platform that processes three fields. On the platform there are three stages of compression with three parallel compressors installed at each stage. Neumann's and Fisher's gas is compressed in all three stages but Nash enters only at the third stage. Between stages there is some knockout of liquids. The compression system is illustrated schematically in Figure 18

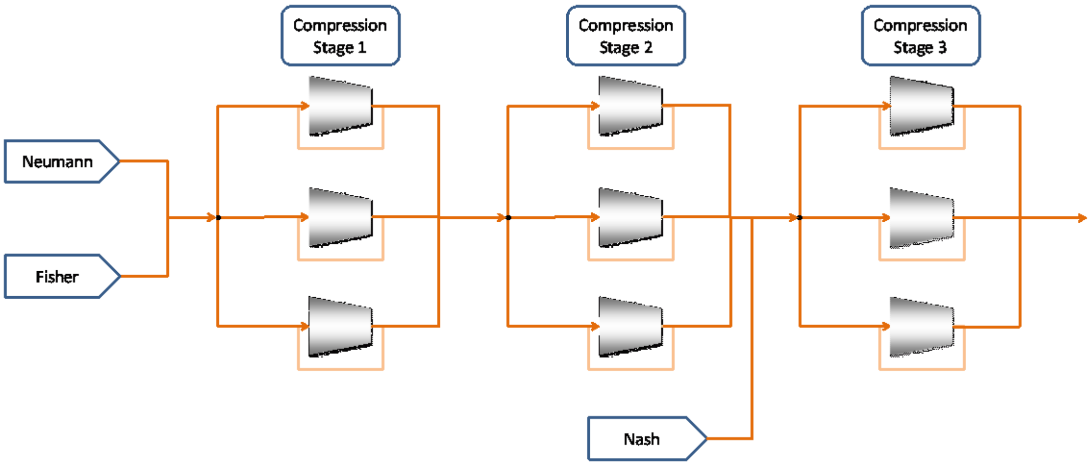

Figure 18 – Schematic of Compression

The compressor performance characteristics are presented in Table 7:

Table 7 Compressor Performance Characteristics

Individual Compressor Attributes	Compression Stage 1	2	3
Nominal Mass Capacity (tonnes/day)	2,055	1,970	2,060
Surge Point (% of Capcacity)	70%	70%	70%
Electrical Energy Consumption (MJ/te)	297	165	246

The capacities and power consumptions of all three sets of compressors do differ. The three fields' throughputs at each compression stage are presented in Table 8.

Table 8 Field Gas Throughputs at Each Compression Stage

Field		Compression Stage 1	2	3
Neumann	tonnes/day	2,029	1,939	1,722
Fisher	tonnes/day	788	747	534
Nash	tonnes/day	0	0	59

The flow of Fisher and Neumann reduces slightly as they progress through the stages reflecting the drop out of condensed liquids. Nash enters the process at the inlet to the third stage and hence has zero flow through the first two stages.

The total electrical power demand associated with compression was calculated in the allocation system using the above parameters and total gas throughputs at each stage. Compression was one component of the total fuel consumed on the platform along with export oil pumping, water injection etc. for which similar power calculations were performed. The actual fuel consumed was then divided between these various components in proportion to the estimated power demands.

The fuel associated with each stage of compression was then allocated in proportion to each field's mass throughput at that stage. Using the methodologies described in Sections 3.4 and 3.5 the Shapley value and nucleolus can be calculated for the compression fuel consumption. The actual allocated quantities are compared with those according to the Shapley value and nucleolus in Table 9 and Figure 19.

Table 9 Comparison of Compression Fuel Allocation Methods

		Neumann	Fisher	Nash
Proportional	(MJ/d)	1,470,911	529,224	18,762
Shapley Value	(MJ/d)	1,121,241	777,017	120,640
Nucleolus	(MJ/d)	1,126,720	774,906	117,271
Differential from Proportional				
Shapley Value	%	-24%	47%	543%
Nucleolus	%	-23%	46%	525%
Differential from Proportional				
Shapley Value	(MJ/d)	-349,671	247,792	101,878
Nucleolus	(MJ/d)	-344,191	245,682	98,509
Approx cost differential				
Shapley Value	$/d	-2,498	1,770	728
Nucleolus	$/d	-2,459	1,755	704
Per year				
Shapley Value	$/y	-911,641	646,030	265,611
Nucleolus	$/y	-897,355	640,527	256,828

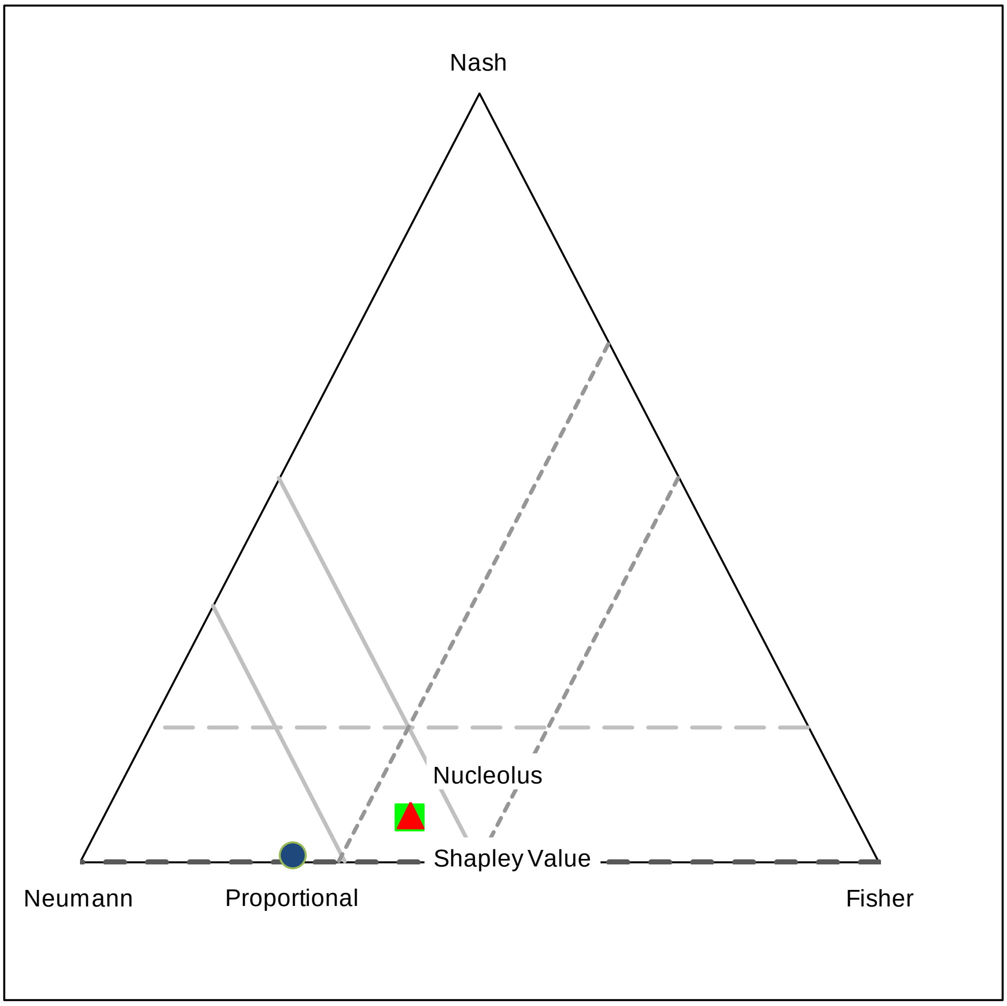

Figure 19 – Comparison of Compression Fuel Allocation

As can be seen the proportional allocation lies outside the core and illustrates that Neumann is disadvantaged. Inspection of the data reveals that it would incur less fuel gas if it was compressed on its own: in effect it is subsidizing both Fisher and Nash's compression costs. Both the Shapley value and nucleolus appear to provide more equitable (almost identical) solutions.

The cost impacts[6] of the three methods are also compared in Table 9. Though small on a daily basis, the impact accumulates to hundreds of thousands of dollars over a year. Another impact to consider is the CO_2 emissions attributed to each field from fuel.

5.2 Allocation of the Effect of Commingling

A concern that is expressed sometimes when a new field is tied back to an existing process is the impact it may have on the existing fields' allocated quantities. In particular if the new entrant has a significantly different composition to the existing fields, it can change the gas oil product split.

If a relatively lean field with a high GOR is tied back to a facility in which relatively low GOR fields are being processed, the lean field will tend to strip components from the oil to the gas phase. This may result in a small but significant reduction in allocated oil production to the incumbent fields.

The following real world example, involves just such a scenario, a relatively high GOR lean field Neumann, is being tied back to an existing offshore platform already processing two low GOR fields, Fisher and Nash.

The allocation is mass based and a simulation model is used to estimate the exported oil production from each field, in the commingled mixture of all the fields, at a component level. The measured oil export is allocated to the Fields in proportion to these estimated quantities.

Using the same model the oil production for all the various combinations of Fields producing can be calculated and incremental impacts calculated at a component level. The allocation according to the Shapley value and nucleolus can be calculated and these are compared with the actual allocation results in Table 10, Figure 20 and Figure 21.

6 Based on a fuel gas GCV of 50 MJ/kg, generator efficiency of 25% and a fuel price of $0.75/therm.

Table 10 Effect of Commingling Oil Allocation

		Neumann	Fisher	Nash
Proportional	(te/d)	2,375	2,964	533
Shapley Value	(te/d)	2,117	3,172	584
Nucleolus	(te/d)	2,116	3,172	586
Differential from Proportional				
Shapley Value	%	-11%	7%	10%
Nucleolus	%	-11%	7%	10%
Differential from Proportional				
Shapley Value	(te/d)	-258	207	51
Nucleolus	(te/d)	-259	207	52
Approx Cost Differential (at $75/bbl)				
Shapley Value	$/d	-143,459	115,101	28,358
Nucleolus	$/d	-144,018	115,035	28,984
Per Year				
Shapley Value	10^6 $/y	-52.4	42.0	10.4
Nucleolus	10^6 $/y	-52.6	42.0	10.6

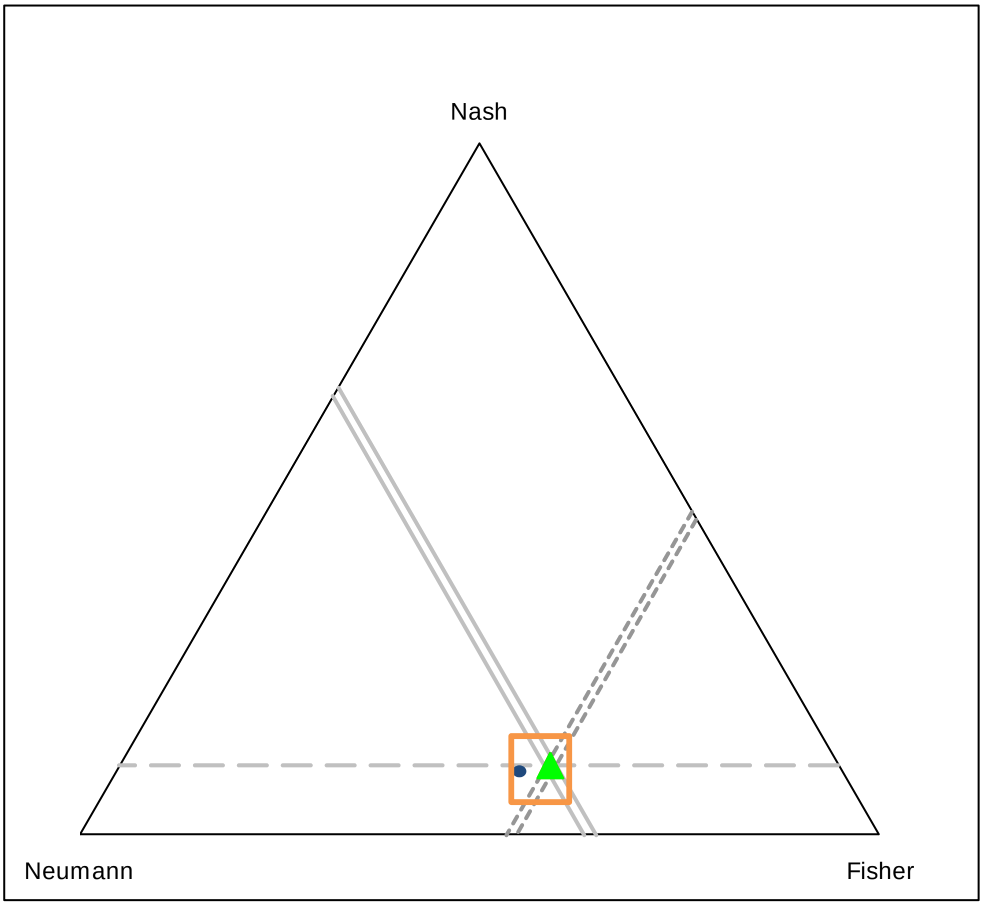

Figure 20 – Effect of Commingling Oil Allocation

The changes in the allocation incurred by the three methods are small compared to the total oil production. Hence the square region is exploded in Figure 21.

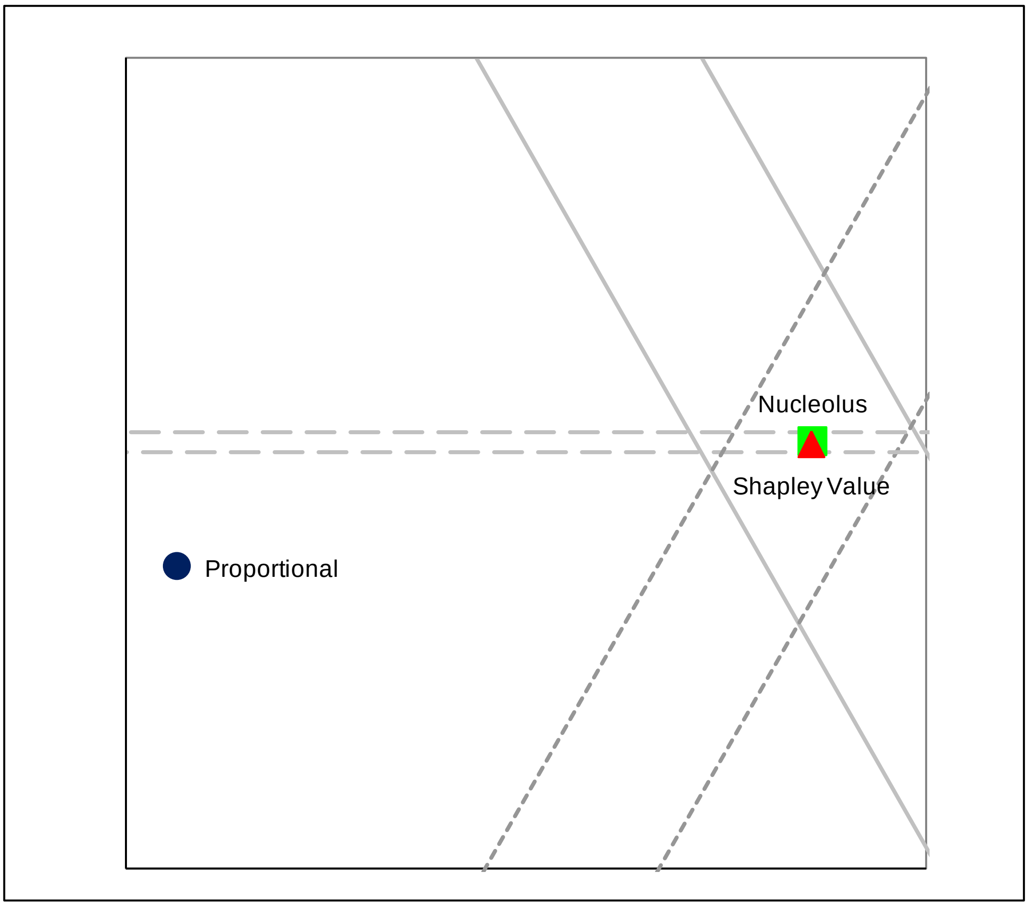

Figure 21 – Effect of Commingling Oil Allocation

Both the Shapley value and nucleolus provide more equitable solutions, which lie in the Core, than the proportional approach. Indeed with the proportional approach, not only has Neumann stripped components out of the oil phase into the gas phase it has increased its oil allocation compared with its stand-alone case and both Fisher and Nash are allocated less oil than either their stand-alone or joint commingled case. Assuming both Fisher and Nash's components are more valuable to them in the oil phase rather than the gas, the introduction of Neumann has a doubly negative impact.

However, adoption of the Shapley value or nucleolus in preference to proportional allocation provides a much more equitable share of the loss of components from the oil to the gas phase. They still afford the use of the mass component based allocation with the use of shrinkage factors derived from a model.

An alternative sometimes adopted by the incumbent fields when introducing a new entrant is to calculate what they would have been allocated prior to the introduction of the new field and then allocate the new field the remaining oil. This has a number of drawbacks since it does not treat all fields in a consistent manner and is not readily extensible for the introduction of further new fields. In contrast the Shapley value and nucleolus achieve the desired outcome but use a consistent, extensible approach, based on sound mathematical principles.

In terms of approximate oil revenue (based on $75/bbl oil price) the three approaches are compared in Table 10. The changes in oil allocation translate into tens of millions of dollars per year of oil revenue to the fields. These figures are offset by gains and losses in the value of the gas allocation. However, it can be concluded that these considerations of equitability in the allocation method does have significant financial implications.

5.3 Capacity Restrictions

As a final example of how game theory can be applied, access to processing capacity when restricted is now examined. Consider a gas terminal with a daily processing capacity of 30mcm/d. The terminal processes gas from two users, Neumann and Fisher, which have booked a daily capacity of 21 and 9 mcm/d respectively. How should the processing capacity be split between Neumann and Fisher if there is a capacity restriction at the terminal?

This could be in proportion to their booked capacities and this is the normal approach adopted in practice. The 'Contested Garment Rule' highlighted in Section 2.1 provides an alternative solution to such a claims problem from proportional allocation. The general rule is:

Calculate each claimants uncontested portion, that is the amount left over after the other claimant has been paid in full or has been paid all that is available. Give each claimant their uncontested portion of the claim plus half of the excess above the sum of the uncontested portions.

Figure 22 shows the result of applying the contested garment rule as the terminal capacity restriction varies, $0 <= V_T <= 30$ mcm/d.

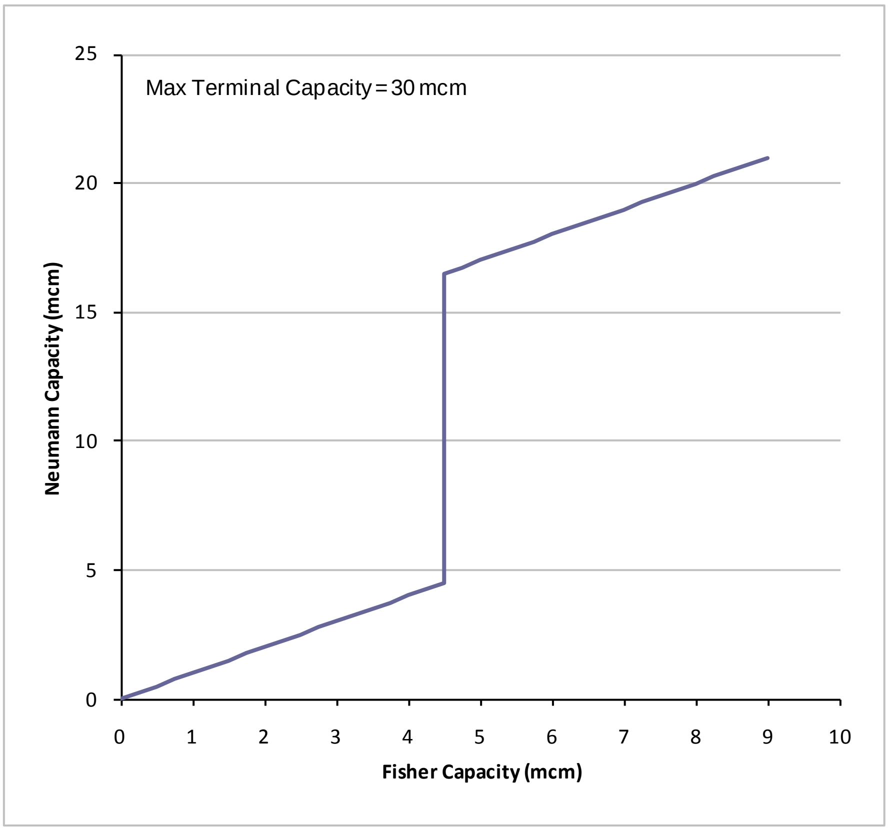

Figure 22 – Division of restricted terminal processing capacity between two users as processing capacity goes from 0 to 30 mcm/d

When the terminal capacity is small, say V_T = 6 mcm/d then Neumann and Fisher's uncontested portions are both zero, so they each receive half of the terminal capacity. This applies for $0 < V_T <= 9$ mcm/d.

For $9 < V_T <= 21$ mcm/d, Neumann's uncontested portion is non-zero so Neumann receives additional processing capacity.

For V_T > 21 mcm/d, both Neumann and Fisher's uncontested portions are non-zero. For a terminal capacity V_T = 24 mcm/d, Neumann and Fisher would receive 18 and 6 mcm/d capacity respectively. In this terminal capacity range the loss in terminal capacity is shared equally between the users.

The general features of this method are that:

- users gain an equal share of terminal capacity, when that is less than the smallest booked capacity;
- users lose an equal share of terminal capacity, when that is more than the highest booked capacity; and,
- for terminal capacities between the user's capacities, the user with lowest capacity receives 50% of their booked capacity until the other user receives 50% of their booked capacity.

The perceived benefit of the Contested Garment approach is that it shares the gains equally among users when the capacity is low and shares the losses equally when the capacity is high. If these are deemed desirable properties then this approach provides an alternative to the proportional allocation.

Although the example described here relates to two users, the method can be extended to any number of users. In that situation a solution could be obtained from a variety of techniques, including Shapley's method. Interestingly the above method is equivalent to calculating the nucleolus and is expanded on in the next section.

6 CONCLUSIONS

It has been demonstrated that when a quantity to be allocated does not vary continuously and directly with a metric such as throughput, proportional allocation does not always produce equitable results.

In many systems allocation in proportion to "something" is adopted just because that "something" is a convenient metric without considering the equitability of the resulting allocation.

Rather than assuming a method is equitable, it is more valuable to consider what desirable properties a method should have that would render it to be deemed equitable. Such features include all users receiving the benefits of sharing costs.

Two alternative methods have been presented: the Shapley value and the nucleolus. These have been developed from the science of co-operative game theory. Such methods have been adopted in many industries to allocate costs. In this paper these methods have been applied to real oil and gas allocation systems and have been shown to provide more equitable allocation results than the incumbent proportional approach.

A 'perfect' solution to an allocation is unattainable. A solution concept must be chosen on the basis of what properties are considered desirable and what counter-intuitive examples are to be avoided.

Finally, though game theory has been formally developed only recently, considerations of equitability are very ancient indeed as has been illustrated in the writings of Aristotle and the Jewish Rabbis. To conclude, a final example provides a fascinating convergence of the ancient ideas of equitability and modern game theory. This was another problem from the Talmud describing how claims on an estate should be divided between three claimants who have claims of 100, 200 and 300 respectively. This is illustrated in Table 11:

Table 11 Division of Estate from the Talmud

		Claim		
		100	200	300
Estate	100	33⅓	33⅓	33⅓
	200	50	75	75
	300	50	100	150

The logic of these divisions puzzled scholars for centuries. What was apparent was that if the estate is "small" it is divided equally and if "large" sized it is divided proportionately. However, the 200 estate division is harder to fathom and in fact was thought to be due to a transcription error until two game theorists provided a beautiful answer in 1985 [8]: it is in fact the nucleolus.

Though the sages of the Talmud would have not known anything about co-operative game theory, let alone the nucleolus, the example illustrates that the principles underlying the nucleolus are very ancient indeed.

7 NOTATION AND ABBREVIATIONS

BOE	Barrel of Oil Equivalent
GCV	Gross Calorific Value
GOR	Gas Oil Ratio
TVA	Tennessee Valley Authority
V_{Cap}	Compressor capacity
V_{Fisher}	Fisher field gas production
V_{Nash}	Nash field gas production
$V_{Neumann}$	Neumann field gas production
V_T	Total gas throughput, terminal processing capacity

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