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Cost Benefit Analyses in the Design of Allocation Systems

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1 INTRODUCTION

This paper considers issues pertaining to the costs of the implementation of measurement systems, e.g. to reduce uncertainty, against the benefits accrued by a reduction in exposure to loss of revenue in allocation systems. Statistical based techniques are presented to assess the risk of loss of revenue.

In Section 2 these issues and methods are discussed and illustrated with simplified theoretical examples. The discussion is principally in terms of flow meters but the issues can equally be extended to any measurements used as inputs to allocation.

Indeed in Section 3 the techniques are applied to data from a real system in which the cost savings accrued from a reduction in compositional sampling frequency were compared with the potential impacts on the allocation system.

2 COST BENEFIT ANALYSIS ISSUES AND CONCEPTS

2.1 Introduction

This section explores the basis underlying cost benefit analysis associated with measurement requirements for allocation.

In Section 2.2 a simple example system is introduced which is utilised to illustrates some of the ensuing concepts explored in Sections 2.3 to 2.7:

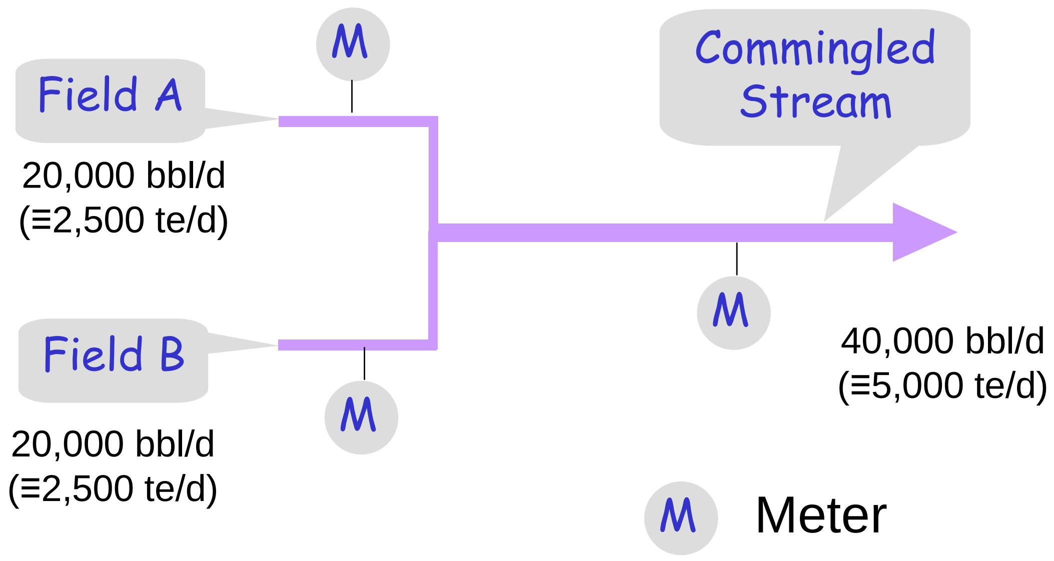
- Systematic allocation bias resulting from differences in meter uncertainty
- Impact of meter uncertainty on the ability to detect meter bias
- Impact of meter uncertainty on allocation uncertainty
- Exposure to loss (risk aversion)
- Techniques to evaluate exposure to loss

These issues are explored and the considerations serve to inform the approach to cost benefit analysis presented in this paper. Finally in Section 2.8 a simplified cost benefit analysis is performed, to illustrate some of the concepts.

2.2 Example System

Consider the simplified example below.

Figure 1 – Simplified Schematic Two Streams Commingling



The flow rates are presented in barrels/day and tonnes/day. For the purposes of the example, the Fields produce similar oils such that their densities are the same and when commingled their standard volumes are additive[1]. This renders the cost benefit analysis calculations more transparent since oil revenue is generally in terms of $/barrel.

The allocation system is a proportional one in which the metered export product is allocated in direct proportion to the metered flow from each Field.

In this example consider the case when the commingled (or Export) stream and Field A's flow are measured to fiscal accuracy: ± 0.25%. What level of accuracy is required for the Field B meter? In reality there is not a continuum of meter uncertainties that could be installed but a number of distinct meter alternatives that can be compared – this is the approach adopted in the simplified cost benefit analysis in Section 2.8.

It might be argued that since the uncertainties in the meters are normally distributed, an individual meter could be over or under reading with equal probability and therefore it doesn't matter how good Field B's meter needs to be. Any gains and losses will even themselves out over a period of time and the cheapest meter should be installed irrespective of its quality. This assumption is not strictly true and is discussed in the next section.

[1] In a real allocation system it would be desirable (generally) to allocate on a mass basis as liquid volumes are not normally additive. Mass is conserved whereas volume generally isn't.

2.3 Allocation Bias

The field with the poor quality meter (higher uncertainty) will be systematically under-allocated product. At first sight this may appear counter-intuitive but consider the example above where the Export and Field A stream meters have a very low uncertainty, negligible in fact when compared to the Stream B meter. Say that each field is actually producing consistently 20,000 bbl/d each. We would observe that Stream A would meter almost exactly 20,000 bbl/d and the total measured Export would be almost precisely 40,000 bbl/d. However Stream B's meter say has a ±10% uncertainty so it is measuring flows in the typically in the range 18,000 to 22,000 bbl/d but over a period of time it is averaging 20,000 bbl/d. On a day when it measures 18,000 bbl/d the allocation to A and B would be:

- Stream A allocated 21,053 bbl
- Stream B allocated 18,947 bbl

However, the next day, B's meter reading swings to 22,000 bbl/d (average over the 2 days is 20,000 bbl/d) and the allocation is:

- Stream A allocated 19,048 bbl
- Stream B allocated 20,952 bbl

So totalising over the 2 days:

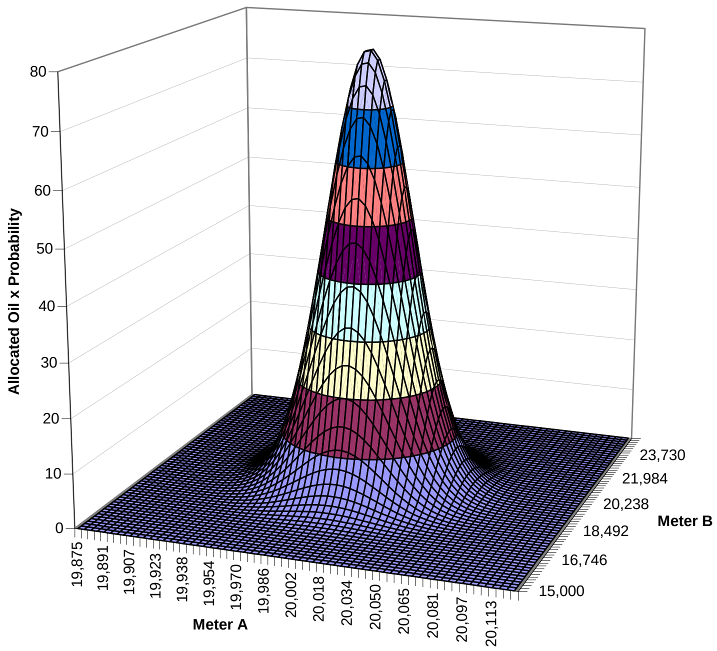
- Stream A allocated 40,100 bbl
- Stream B allocated 39,900 bbl

Stream B has been under allocated by 0.25%. This may not appear to be a large percentage but it is systematic and refutes the claim that "things will even themselves out over a period of time". Interestingly justice has been done as the stream that has invested in the better quality metering as at an advantage.

In reality the meter readings would be over-under reading with probabilities dictated by the normal distribution and degree in accordance with their uncertainty. Based upon these distributions an expected value of the allocated oil to Field A and B may be calculated.

In effect this calculation takes every possible value that the Field A and B meter readings could have, calculates the allocated quantities, and multiplies the result by the probability of its occurrence. The sum of these probability weighted values is the expected allocation result based on the probabilities. This is presented pictorially in Figure 2 for Field B's allocation:

Figure 2 –Field B Probability Weighted Allocated Oil



The horizontal axes are the range of A and B meter readings, covering ± 5 standard deviations around the average value of 20,000 bbl/d. This covers in excess of 99.9999% of all the possible values each meter could read consistent with its uncertainty. (It should be noted that the scales of these axes are different).

Each point on the surface represents a quantity allocated to Field B corresponding to the values of Meter A and B readings multiplied by the probability of those two meter readings occurring. The total volume under the surface is the "expected" allocated quantity and this represents the average allocated quantity over a period of time.

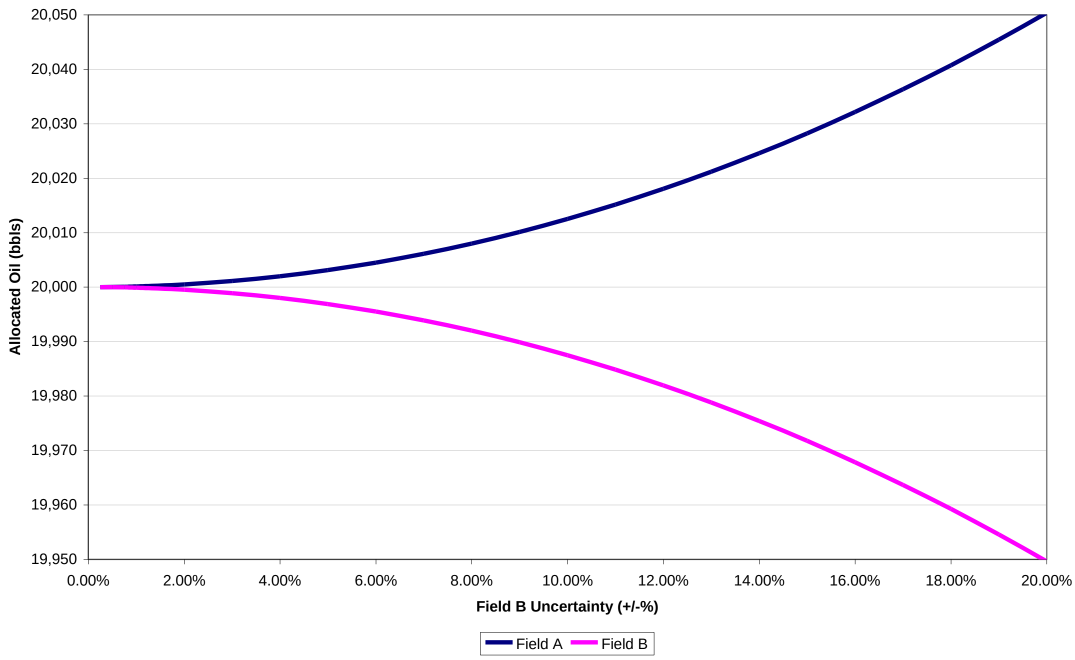
The most probable allocation occurs when both meters read 20,000 bbl/d and this is where the apex of the surface occurs. As the readings move away from the average the probability diminishes to almost zero within 2 standard deviations. The surface is not actually symmetrical in all planes and is weighted towards an under-allocation to Field B.

For the case where Field A meter is ±0.25% and Field B's is ±10% uncertainty, the expected allocation to the two Fields is:

- Stream A allocated 20,012.5 bbl
- Stream B allocated 19,987.5 bbl

Section 4.2 presents the mathematical derivation of the expected under/over allocation as a result of differences in metering quality. To illustrate the impact of Field B meter uncertainty on both Fields' expected oil allocation, the under/over allocations are plotted as a function of its meter B uncertainty in Figure 3.

Figure 3 – Expected Oil Allocation – Variation in Meter B Uncertainty



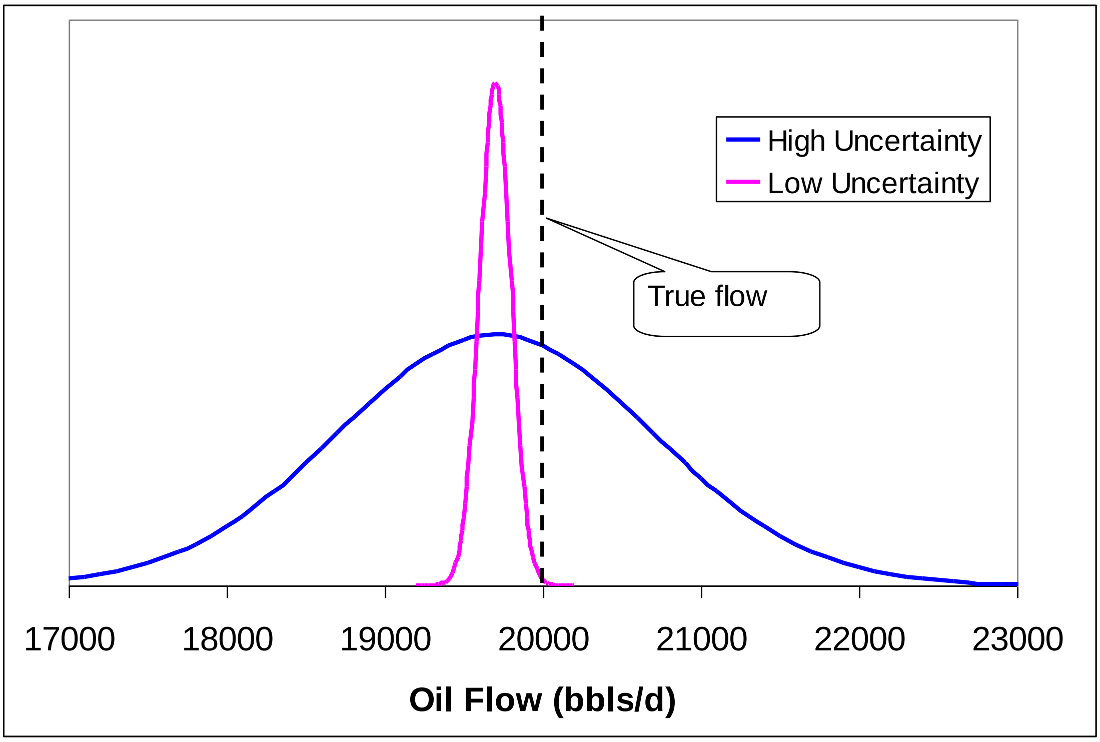
This bias in the allocation is small but it is systematic and occurs as a result of the mathematics of the equations. In the above example, at 10% meter uncertainty for Field B, the expected under-allocation is 12.5 bbl/d which is worth approximately $625/d (assuming 50 $/bbl oil price). Over a year this translates into nearly a quarter of a million dollars.

Though a small effect it is important to understand when apparently reasonable assumptions are not true.

2.4 Meter Bias Detection

A second reason meters with relatively high levels of uncertainty are not desirable is that they can mask systematic bias as illustrated in Figure 4.

Figure 4 – High and Low Quality Meter Uncertainty Distribution in Realtion to Bias



The true meter reading is 20,000 bbl/d but both meters under read by 300 bbls/d. With the lower quality meter (higher uncertainty) the problem is not so easy to detect since the bias is still located in the central broad peak of its probability envelope. In contrast, the bias lies beyond the standard uncertainty confidence level and would be more apparent.

2.5 Impact on Allocation Uncertainty

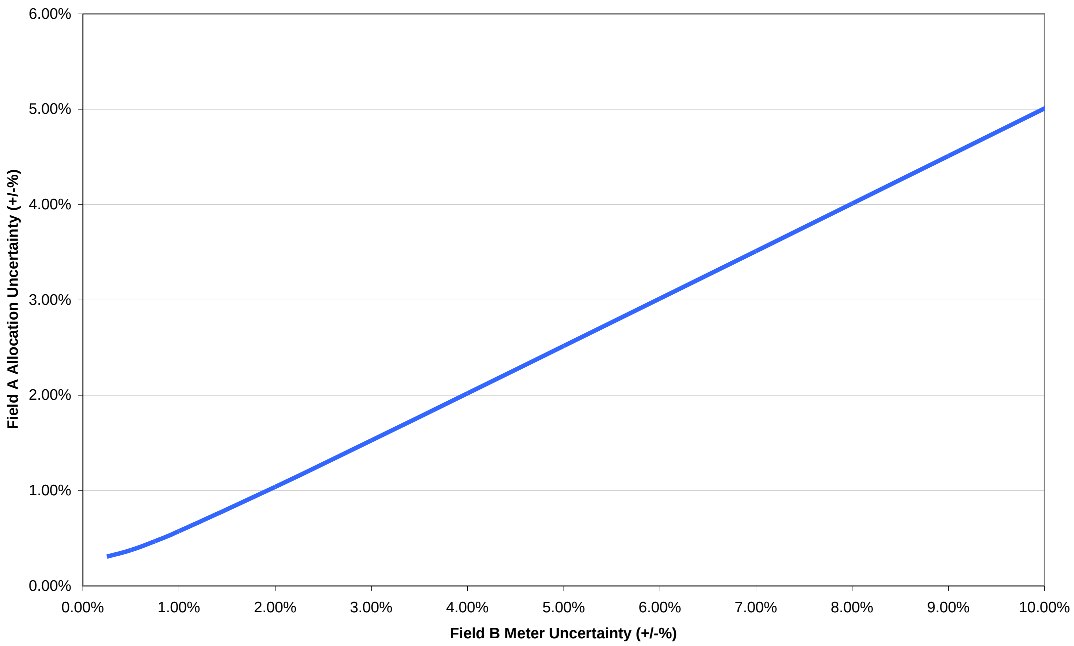
In proportional allocation systems the uncertainties in the contributing stream flows have impact on each others allocated quantity uncertainty. Consider the example above, where Field A and the Export meter uncertainty are ±0.25%. Field B's meter uncertainty is ±10%, at the example flowrates the uncertainties in the allocated quantities are:

- Stream A allocated oil uncertainty ±1,000 bbl, ±5%
- Stream B allocated oil uncertainty ±1,000 bbl, ±5%

These uncertainties were calculated analytically using propagation of uncertainties as described in the GUM [1].

Both streams have the same allocation uncertainty but for the case of Stream A, the uncertainty is much greater than its individual meter uncertainty. The variation in allocation uncertainty with Stream B meter uncertainty is presented in Figure 5.

Figure 5 – Field Allocation Uncertainty with Variation in Meter B Uncertainty



Another option would be not to install a meter at all for Field B and allocate by difference. This reduces Field A's allocation uncertainty to ±0.25% (i.e. just equal to the Field A meter uncertainty) and Field B's uncertainty is ±0.56%. However, there are other issues with by difference allocation which have to be considered. For example there is no quality check built into the allocation in that if Field A meter develops a problem this might not be detected so easily compared with the proportional allocation system in which the sum of the Field meter flows should be close to the total metered export (within the uncertainties). Also if Field B flow reduces then its uncertainty rises sharply, at 1000 bbl/d it is over ±7% and at 100 bbl/d it rises to over ±70%.

2.6 Exposure to Loss (Risk Reduction)

One of the key drivers in selection of meter is the risk of exposure to loss.

For one moment, if the issues surrounding meters with higher uncertainties (raised above in Sections 2.3 to 2.5) are put to one side, then from a pure cost perspective shouldn't the cheapest meter always be installed?

From a purely probabilistic viewpoint the increased uncertainty introduced by installing low quality metering is just as likely to result in a gain as a loss to one Field (compensated by an equal and opposite gain or loss in the other Field) because the meter is just as likely to under-read as over-read.

This might be a reasonable approach if the Field owners had many such systems and the value of the product was low since on the average over all the systems it is most probable that the allocation results would even themselves out. In oil and gas systems the value of the product is normally high compared with the meter costs and it is unlikely that an investor in a Field would have such a vast portfolio of systems that he would be relaxed that all the gains and losses would even themselves out.

In the simplified example above, 10,000 bbl/d represents a revenue stream of $500,000/d so an improvement in allocation uncertainty of 1% would reduce the uncertainty in allocated revenue by $5,000/d. This could be a gain or a loss though to an individual Field but that would be compensated by an equal gain or loss to the other Field(s). The extra cost of better quality metering is a cost to the system as a whole and therefore a guaranteed expense but it buys more certainty in the allocated quantities. In effect the investment in the improved metering is buying insurance against loss of revenue. The study of attitude to risk in decision making is termed Preference or Utility, Theory. An example serves to illustrate the concept:

Imagine you are presented with the choice of being given $1,000 or being entered into a lottery, the outcome of which depended on the toss of a coin, in which you stood to win $2,000 or nothing with equal probability. Based on the probabilities the expected average outcome of both choices is $1,000. However, most people would tend to select the guaranteed $1,000.

However, if the game was changed slightly so that the guaranteed quantity was reduced to $900 then on probabilistic grounds alone you should enter the lottery. However, it is likely a lot of people would still take the guaranteed $900. In effect these people are giving up $100 to insure against receiving nothing.

If the guaranteed quantity was reduced further then there would come a value when you decide that it was worth risk to play the lottery. The reduction in the guaranteed quantity to this point is the cost you are prepared to pay to insure against a loss.

The essential point is that some money has effectively been traded to insure against a loss. Translating this to the cost benefit analysis, how much are you willing to invest in a meter for Field B to reduce the exposure to loss of revenue caused by the uncertainty in the meter. The difficulty is that the amount to be invested is subjective and depends on a number of factors:

- The amounts involved; if in the above lottery example the guaranteed amount was $1 and the maximum win was a $2, it is much likely you would gamble since you could probably afford to lose $1. If the guaranteed amount was $1,000,000 it is unlikely you would be willing to gamble that for the chance of winning $2,000,000.
- Your attitude to risk, some people and organisations are more less risk averse than others.

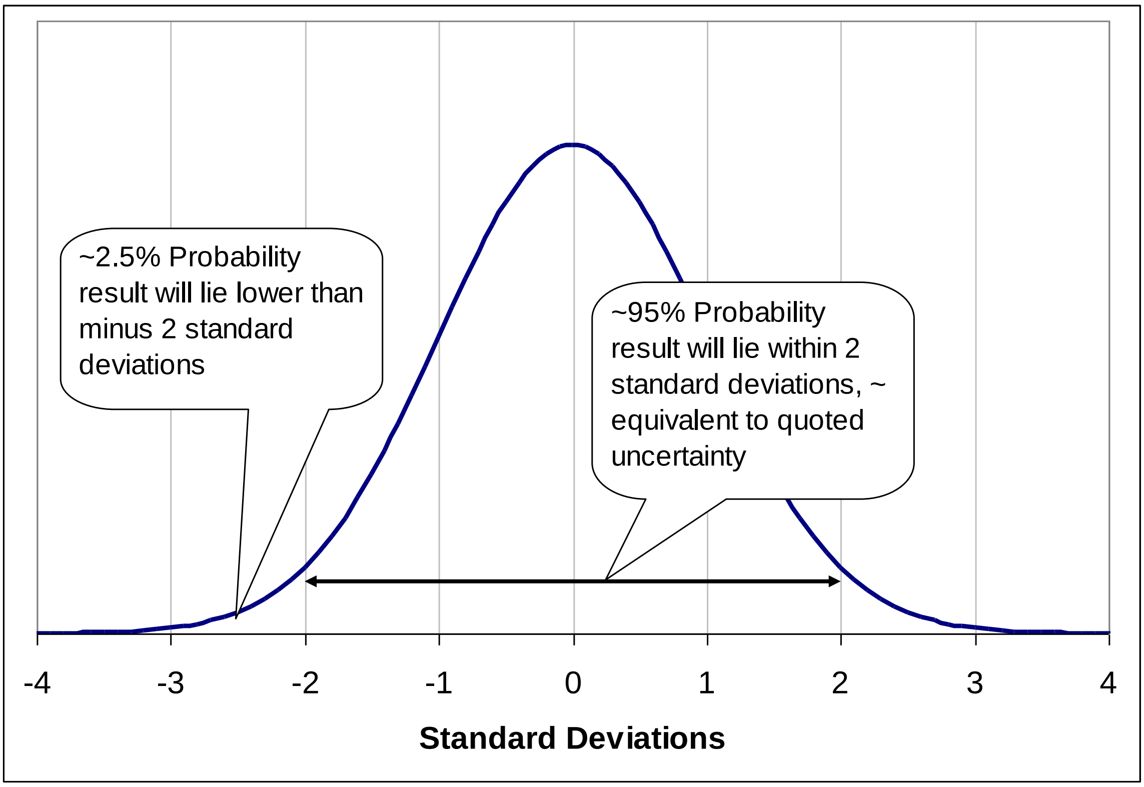
The next section describes two possible methods to evaluate the exposure to loss.

2.7 Evaluating Exposure to Loss

Allocation Uncertainty Approach

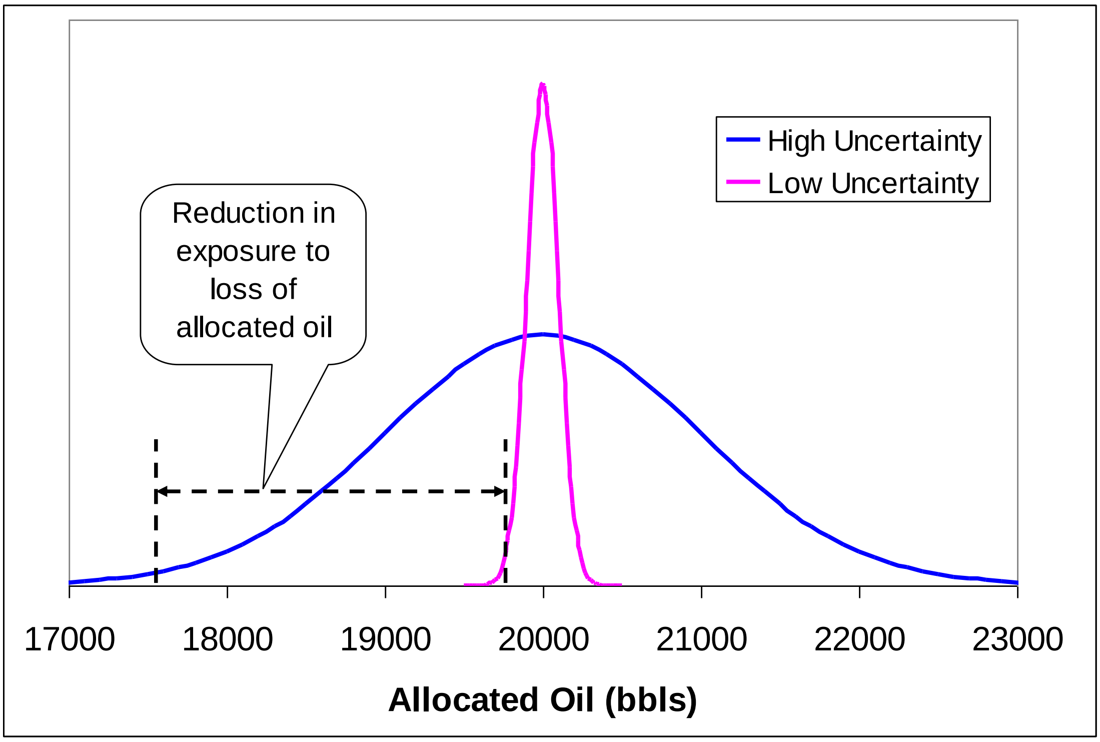
This involves calculating the Field allocation uncertainty associated with the different meter options. The minus side of this uncertainty will be the exposure to loss at the 97.5% confidence level, i.e. there is a 2.5% probability that the loss will lie beyond the minus uncertainty band. This is illustrated in Figure 6.

Figure 6 – Distribution in Allocation Uncertainty



The allocation uncertainty can be calculated for each meter option and converted into an equivalent revenue quantity. The difference in revenue uncertainty associated with the two options then represents the reduction in exposure to loss (over Field life) and this can then be compared against the difference in meter costs. This is illustrated in Figure 7.

Figure 7 – Reduction in Loss Exposure Based on Allocation Uncertainty



These figures were calculated at the 97.5 % confidence level. The answer could be different based on different confidence levels.

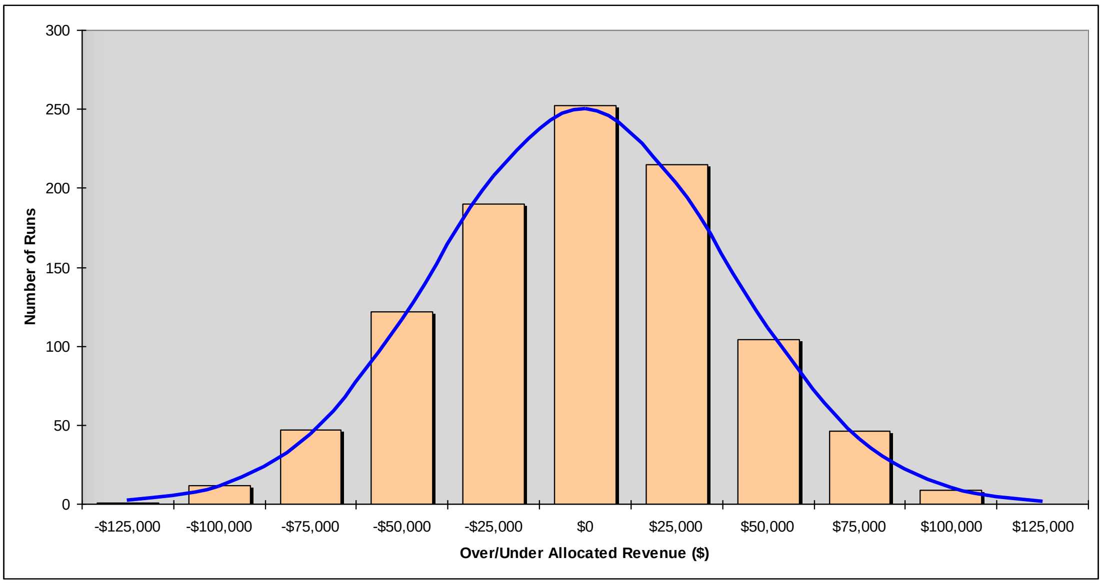
The next approach integrates the risk of loss over a range of confidence levels.

Integrated Risked Exposure Approach

By analysing the impact of the variations in Field B's metered flow, due to its uncertainty, on the allocation results the mis-allocation of revenue can be calculated.

Using the example above the impact of variations in the meter reading on the allocated revenue has been analysed using a Monte Carlo simulation. In each run of the simulation, the meter flow was varied in accordance with appropriate standard deviation figures and the allocated revenues collated. The results of such a Monte Carlo simulation for Field B are presented in Figure 8.

Figure 8 – Distribution in Field B Revenue



1000 runs were used to generate the data in the above chart. The histogram bars represent the number of runs in which the allocated revenue lay in the bandwidth indicated on the horizontal axis – (the figures on the x-axis are the midpoints of the bands). The values on the axis refer to the difference between daily allocated revenue in the run compared with the average value, i.e. the over/under allocation of revenue compared with the average.

The blue line is a plot of the normal distribution curve with the same average and standard deviation; the curve demonstrates that the allocated revenue is normally distributed.

On any individual run there is a chance that the allocated revenue could be anywhere along the x-axis but the probability diminishes the further from the mean. Because the revenue is normally distributed the probability or risk associated with any individual under- or over-allocation of revenue figure can be calculated.

It is possible to multiply each lost revenue figure by its individual probability of occurrence. These can then be summed to give a total risked lost revenue figure. This is, in effect, the Integrated Risked Exposure to mis-allocated revenue and is calculated by the following equation:

$$L_B = \frac{UR_B}{\sqrt{8\pi}} \qquad (1)$$

The derivation of this equation is presented in Section 4.1.

The difference in the risked loss exposure for the two metering options can then be compared with the difference in meter costs as described above for the Allocation Uncertainty approach.

The two methods give different answers, with the Allocation Uncertainty approach being more conservative. There is no right or wrong method because the approach adopted depends on attitude to risk aversion. They are just two possible methods that provide a degree of auditability but the final decision should be based on engineering judgement, considering all the factors discussed in the above sections.

2.8 Simplified Cost Benefit Analysis

The two methods are applied to a simplified example in this section. Using the throughputs and meter uncertainties presented in Section 2.2 a comparison is made between installing an equivalent fiscal quality meter for Stream B (± 0.25%) versus an allocation standard meter with an uncertainty of (± 5%).

The figures presented are fictitious but intended to be roughly representative.

The cost of a fiscal quality meter, with attendant proving facilities, etc., has been estimated to be $5,000,000. The yearly maintenance costs associated with maintaining its accuracy have been assumed to be $50,000 per year.

The cost of the allocation quality meter has been assumed to be $500,000. The meter maybe the same type as the fiscal quality meter but it won't have all the proving, spares, etc. The yearly running costs have been assumed to be negligible.

The analysis is based on a 10 year life at an oil price of $50/bbl.

The figures for the case where a fiscal quality meter is installed for Field B is presented in Table 1.

Table 1 – Cost Benefit Analysis Field B Fiscal Meter

		Field A	**Field B**	**Export**
Flow	bbl/d	20,000	20,000	40,000
Field B Fiscal Meter				
Meter Uncertainty (Relative)	±%	0.25%	0.25%	0.25%
Allocation Uncertainty (Relative)	±%	0.31%	0.31%	
Loss exposure at 95% confidence level (Field Life)	$	$11,175,797	$11,175,797	
Loss exposure integrated (Field Life)	$	$2,229,249	$2,229,249	
Meter Installation Cost	$		$5,000,000	
Meter OPEX (Field Life)	$		$500,000	
Total Cost	$		$5,500,000	

The allocation uncertainty for both fields is ±0.31% and at the quoted flows this equates to a loss exposure of over $11 and $2 million for the allocation uncertainty and integrated risked approaches respectively.

Performing the same analysis for the allocation quality meter produces the results presented in Table 2.

Table 2 – Cost Benefit Analysis Field B Allocation Quality Meter

Field B Allocation Meter		Field A	Field B	Export
Meter Uncertainty (Relative)	±%	0.25%	5.00%	0.25%
Allocation Uncertainty (Relative)	±%	2.52%	2.52%	
Loss exposure at 95% confidence level (Field Life)	$	$91,818,541	$91,818,541	
Loss exposure integrated (Field Life)	$	$18,315,149	$18,315,149	
Meter Installation Cost	$		$500,000	
Meter OPEX (Field Life)	$		$0	
Total Cost	$		$500,000	

The allocation uncertainties for both Fields have now increased to ±2.52%. The poorer quality Field B meter is having a deleterious effect not only on Field B's loss exposure but also on Field A's despite the investment in its fiscal quality meter.

The reduction in meter costs are compared with the increased loss exposure for the two approaches in Table 3.

Table 3 – Cost Benefit Analysis Comparison of Field B Allocation Quality Meter

Cost Benefit Analysis		
Meter Cost Saving	$	$5,000,000
Increase in Loss Exposure at 95% Conf Level	$	$80,642,744
Increase in Loss Exposure integrated	$	$16,085,900

Since the exposure to loss is experienced by both fields it could be argued that the values should be doubled when considering the impacts on the system as a whole and not just Field B.

The two methods produce seemingly largely different loss exposure figures but in fact the two methods provide similar conclusions. To illustrate this, the uncertainty in the Field B meter would need to be reduced to 1.9% to reduce the integrated risk loss exposure to $5,000,000 compared with 0.7% for the 95% confidence level exposure. Similarly a reduction in the Field B meter flow would reduce the exposures and for the Integrated Risk Exposure approach the flow would need to be 3,500 bbl/d to reach the break even point and to around 600 bbl/d for the Allocation Uncertainty approach. The reduction in exposure is not directly proportional to either the flow rate or the meter uncertainty because of the non-linearity in the Field allocation uncertainty.

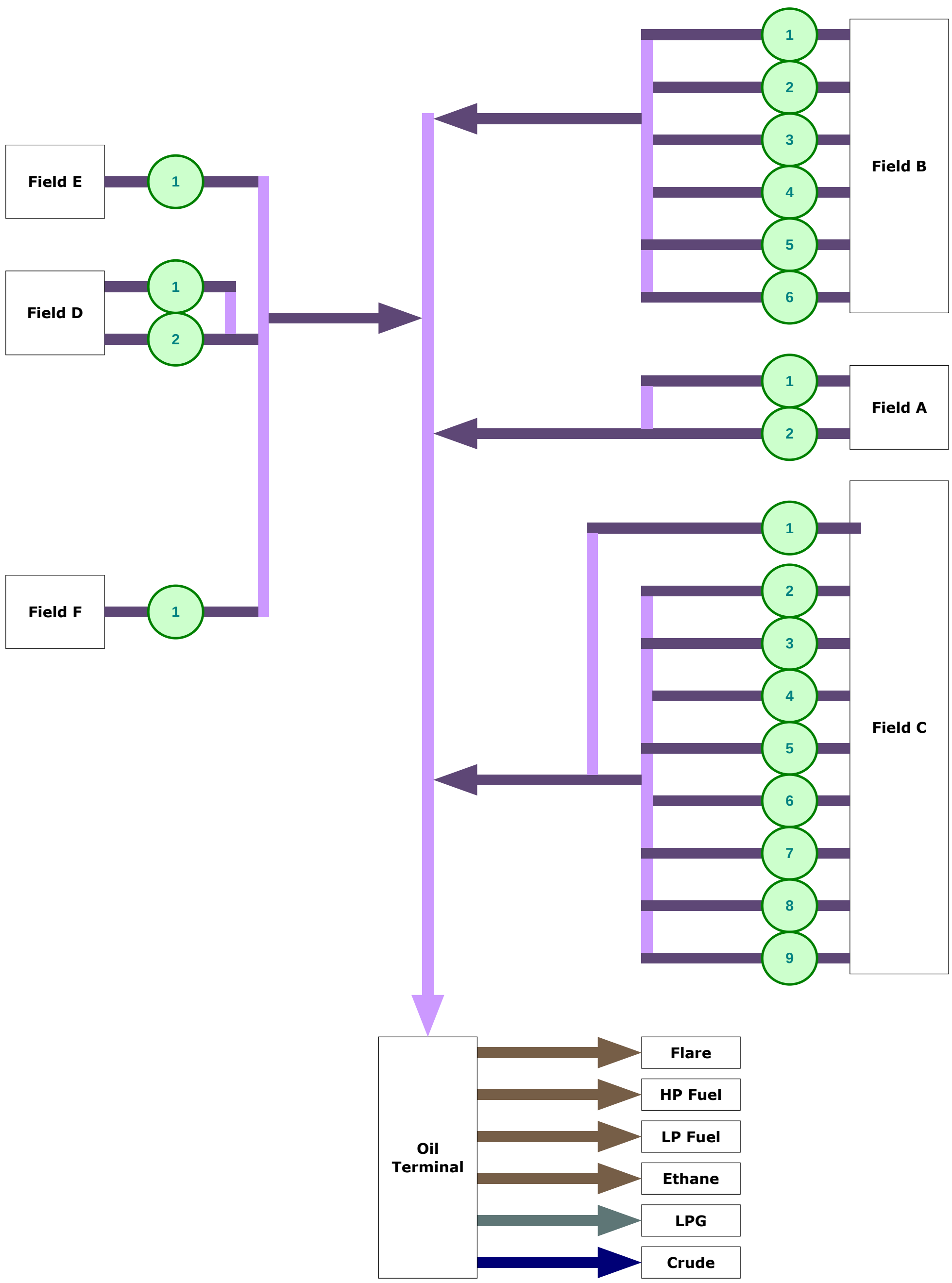
For this simplified example both methods illustrate the exposure to loss is significantly in excess of the cost of the meter upgrade.

3 COST BENEFIT ANALYSIS OF SAMPLING OPEX IN AN OIL SYTEM

The example presented concerns an oil terminal allocation system fed by a number of offshore facilities via a subsea pipeline see Figure 9.

Oil is fed from various offshore facilities (Fields) into a common pipeline and delivered to the onshore terminal. The products from the terminal are allocated to the various Fields based on the quantities they have delivered to the terminal. This allocation is performed at a mass based, hydrocarbon component/fraction level.

Figure 9 – Oil Pipeline Schematic



The compositions of the various streams, associated with each Field, vary considerably ranging from heavier oils to light condensate streams.

An integral part of the allocation system is the sampling and analysis of fluids, which incurs OPEX in the region of £350,000 per annum. With a decline in production through the oil terminal reductions in OPEX associated with the system were being sought.

The Fields' feed Streams are all metered and sampled to determine their composition and flow, as are the onshore products. Weekly samples are taken from each Stream and analysed; these weekly samples are also combined into a monthly sample upon which a detailed assay is performed. However, it was observed that some of the compositions of the Streams delivered into the pipeline appear to remain relatively constant.

The objective of cost benefit analysis was to analyse options to reduce weekly and monthly sampling (and analysis) and hence OPEX associated with the allocation system whilst ensuring the integrity of the system is not compromised and safeguarding against unacceptable exposure to mis-allocation of the terminal products.

For the purposes of this illustration the OPEX savings associated with a reduction in the weekly samples is presented. A similar analysis (not presented) was applied to the monthly assays.

3.1 Sampling and Analysis

For each Stream, ideally samples are collected over a week using flow proportional auto-samplers. This means that for practical purposes the samples collected are completely representative of the fluid that has been produced in that Stream over the sample period. This means that the current integrity of the Allocation system is high as is reasonably practical.

The weekly analysis measurements are used to allocate the crude, LPG and ethane products along with fuel gas and flare at the Oil Terminal.

The cost of sampling was approximately $400 per sample for the weekly samples. Approximately 50 samples per month were taken with a total monthly OPEX of approximately $20,000 per month.

The OPEX was split between the fields in proportion to their throughput:

Table 4 –Allocation of Sampling OPEX

Field	**Sampling OPEX ($)**		
	Allocated OPEX	**Monthly Throughput (tonnes)**	**Percentage of Production**
A	2,290	64,985	11%
B	5,488	155,746	26%
C	8,820	250,281	42%
D	728	20,665	3%
E	397	11,271	2%
F	3,090	87,692	15%

The cost benefit analysis considered the impact of reducing OPEX by obtaining samples on a discontinuous basis; e.g. collect the weekly samples only one week out of four (omitting three weeks of samples). This approach relies on the sample obtained being sufficiently representative of the period over which it is applied in the allocation; hence this introduces risk of mis-allocation of Oil products.

The approach involved a fundamental change in the philosophy associated with the Oil Allocation system, in that sampling would be performed on a discontinuous rather than the current continuous basis. This introduces an element of risk, since it is possible that the assumed constant composition may in fact vary in the un-sampled period; this would result in a mis-allocation of Oil products associated with the Field whose sampling frequency was reduced. Any gain enjoyed by that Field would be exactly balanced by a corresponding loss distributed across the other Fields and vice versa. Hence, any mis-allocation would impact all Fields in the allocation system.

Sources of variation in individual Stream compositions and properties can be attributable to a number of factors, which include:

- Changes in the relative flows of wells from different reservoir zones
- Changes in the composition of the hydrocarbons in the reservoirs from which the Stream is produced, e.g. caused by falling reservoir pressure
- New wells brought on Stream
- Changes in offshore process operating conditions
- Uncertainty in the laboratory sample measurements.

With regard to reducing sampling frequency the desirable behaviour of the measured composition would be:

- The underlying average value is constant for sustained periods, i.e. there is no systematic shift in the value over the period
- Any fluctuations are random about the average value
- The fluctuations are sufficiently small that the savings in reduced sampling and analysis OPEX outweigh any mis-allocation in revenue incurred
- Or, the size of the fluctuations is within the legitimate measurement uncertainty (i.e. within measurement tolerance).

An important aspect of this approach is determining, based on historical data, whether a Stream’s composition does remain essentially constant within acceptable levels of variability. The level of confidence in this assumption increases with the number of consecutive historical analyses over which the composition or property was deemed to be stable. A reduction in sample frequency can then be justified once the desired confidence in the stability of Stream has been established.

The next section (Section 3.2) is concerned with establishing the degree of variability in compositions and examining the associated impact on allocated revenue. This was accomplished by simply reviewing the data and by the application of some statistical methods.

3.2 Compositional Variability

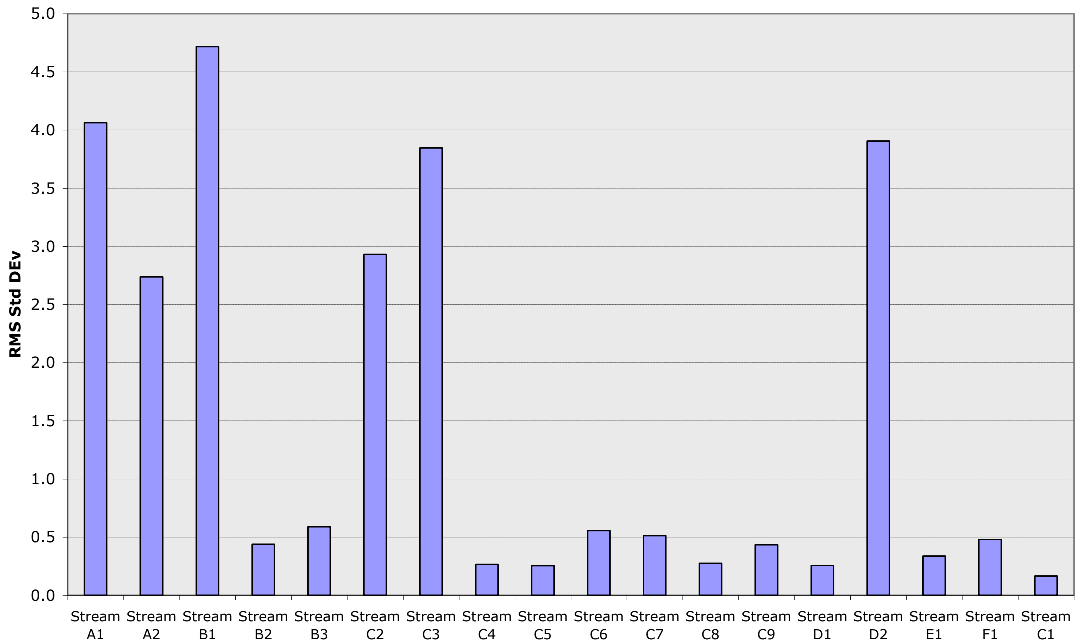
Some statistical analysis was employed to detect the presence of trends in the data along with some sensitivity analysis to estimate the impact of the changing compositions and properties on allocated revenue.

General observations associated with the compositional data were:

- There are two types of Streams: oil and condensate/NGL
- Oil Streams consist principally of C5P (>90%)
- Lighter condensate/NGL Streams have a more evenly distributed spread of components
- The variability in the composition of the lighter condensate Streams is considerably greater than that observed with the oil Streams.
- For the oil Streams, any increase in C5P generally results in corresponding decreases in the majority of the other components and vice versa .

In order to condense the data and directly compare the variability of the various Streams, the deviations associated with all the components in a Stream were pooled to produce a figure that represented the overall variability of a Stream's composition[2]. It is not the absolute pooled standard deviations themselves that are of interest but rather the relative values between the Streams. The pooled standard deviations have been used as a mechanism to determine the most and least compositionally variable Streams and are presented in Figure 10:

Figure 10 – Weekly Compositional RMS Variability



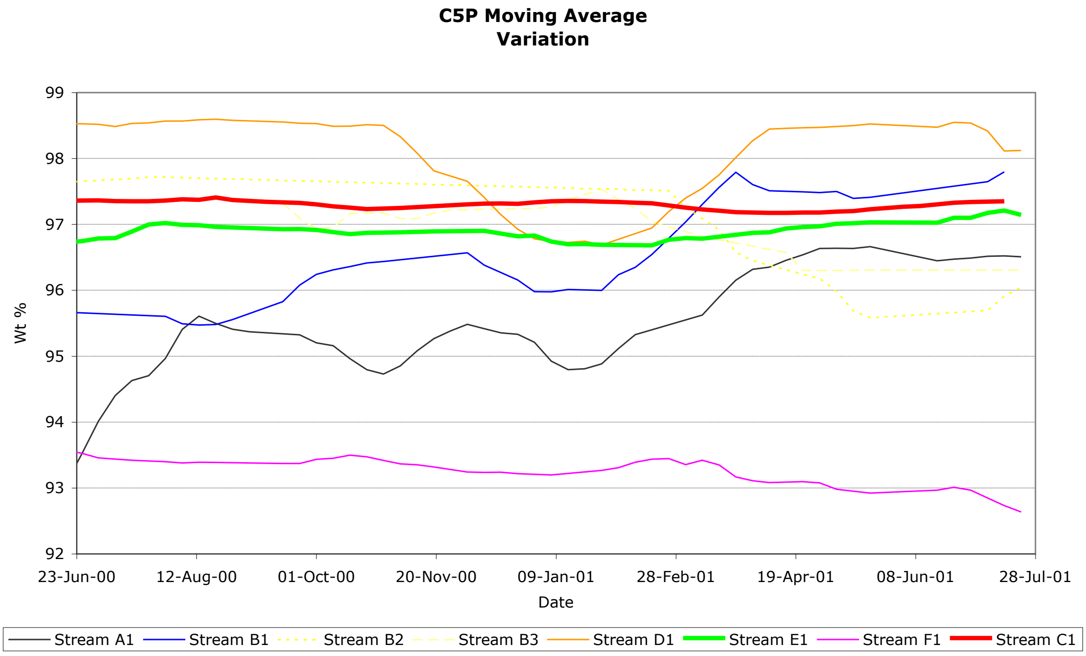
Streams E1 and C1 were focussed on in the statistical analyses because of their low compositional variability. The rationale adopted was that if a case to reduce sampling

[2] The pooled standard deviation was obtained from the root mean square (RMS) of the component standard deviation figures.

frequency cannot be demonstrated for these apparently stable Fields and Streams then the case cannot currently be made for any of the Fields or Streams.

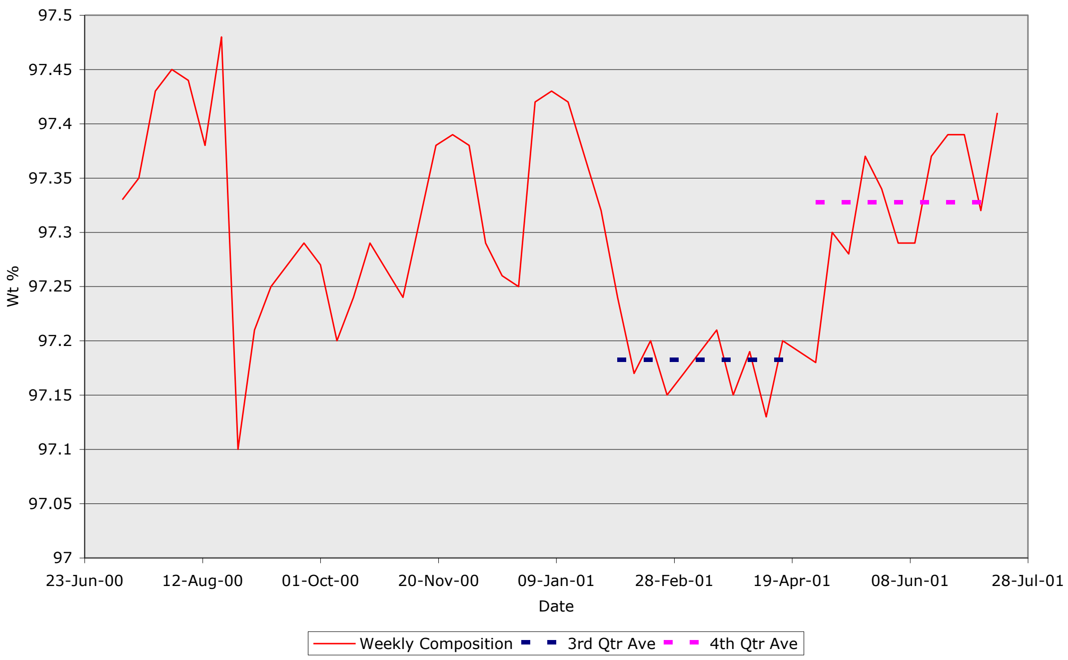
In order to focus on longer-term changes in the various Streams' compositions the dominant C5P component was considered in further detail. To remove some of the scatter observed and identify systematic trends, the 8-week rolling average of the C5P content has been plotted in Figure 11 for several of the streams.

Figure 11 – C5P Component Moving Average



Streams C1 and E1 (emboldened in the plot) do appear comparatively stable but even these streams show evidence of compositional drift. The Stream C1 C5P content (not averaged) is plotted on an expanded scale in Figure 12:

Figure 12 – Stream C1 C5P Variation June '00 to July '01



The red line depicts the actual data values, the blue dotted line is the average value in the third quarter and the purple line the average in the 4th quarter of the period considered. There appears to be a systematic rise in the C5P content of approximately 0.15 wt% between the final two quarters. The question of whether this shift is a systematic change or whether it could have arisen by chance alone, consistent with the typical scatter in the data, has been tested statistically□[3]. The results of the test show that the change in composition is statistically significant.

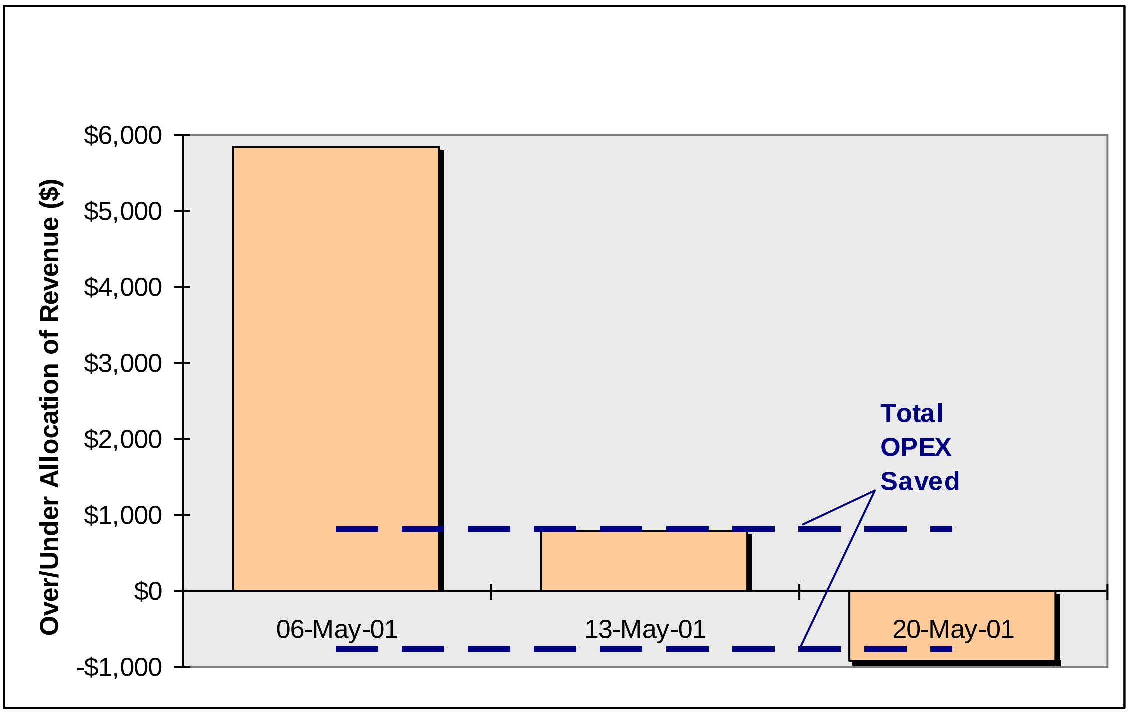
This Stream in particular had been identified as one in which the composition was stable. However, the analysis illustrates that even in this apparently stable stream there are detectable variations in composition and it is not safe to assume any of the streams' composition remained constant.

However, these small changes, though real, may result in little impact on the allocation results. This has been analysed for Field E, which also exhibits relative compositional stability. Field E was selected as it comprises only one Stream (E1) and hence the impact on its allocated quantities was relative simple to determine. This Field has the lowest throughput (see Table 4) and hence impacts on allocated revenue would be the smallest.

[3] The method employed to determine if there is a change in the average value of the C5P content between the 3rd and 4th quarter was a comparison of means for independent samples using a two-tailed Student's t-test. An F test was initially used to determine if the statistical variances in the data for the two periods could be pooled to calculate the standard error in the means. If not, then Satterthwatite's approximation was used to determine the degrees of freedom in the calculation of the standard error. The value of the standard error was compared against the observed difference in the means from the two periods to determine the t statistic value. The confidence level that the two means are different can then be determined from standard reference tables of the t statistic.

To gain an understanding of the impact of reducing the sample frequency, the allocation results were recalculated substituting each individual weekly sample composition, in place of the flow-weighted combined composition, as inputs to the allocation model. The Field E revenue, based on the flow-weighted analysis over the four weeks, was $2,541,018. The flow-weighted analysis comprised three samples taken over the period. Figure 13 shows the impact on allocated revenue of basing the allocation on the individual samples compared with the averaged composition:

Figure 13 – Field E - Impact of Weekly Sample Composition



The histogram bars represent the difference in allocated revenue associated with the individual weekly compositions compared with the combined composition. Hence, the first histogram bar shows that Field E would have been allocated nearly $6,000 dollars more in revenue terms if the allocation had been based on the sample drawn from the first week only. This also means that the other five Fields would collectively have lost $6,000 revenue. Though Field E appears to gain $6,000 based on the first weekly sample, the allocation could have resulted in a loss as can be seen from the chart if the allocation had been based on the 20th May sample.

The OPEX associated with each Stream's weekly sample is $400, hence if two out of the three Field E samples were omitted the total OPEX saved would have been $800 dollars over the month - this value is indicated by the dashed blue lines on the chart. This provides some indication of the relative value of the total OPEX savings in comparison to the potential mis-allocated revenue for the system as a whole.

The impacts presented in Figure 13 are associated with the Field E, whose throughput is the smallest and whose composition is relatively stable. Larger impacts are observed for other Fields, for example a similar reduction Field F's weekly sampling frequency results in mis-allocated revenue of the order of $60,000.

Returning to Stream C1, the 0.15 wt% rise in C5P content described above, incurs an increase in allocated revenue of approximately $15,000 based on its typical throughput of 100,000 tonnes per month.

These figures indicate the significant level of mis-allocation of revenue potentially incurred for even the most apparently stable compositions, compared with modest savings in OPEX afforded by reducing their sampling frequency.

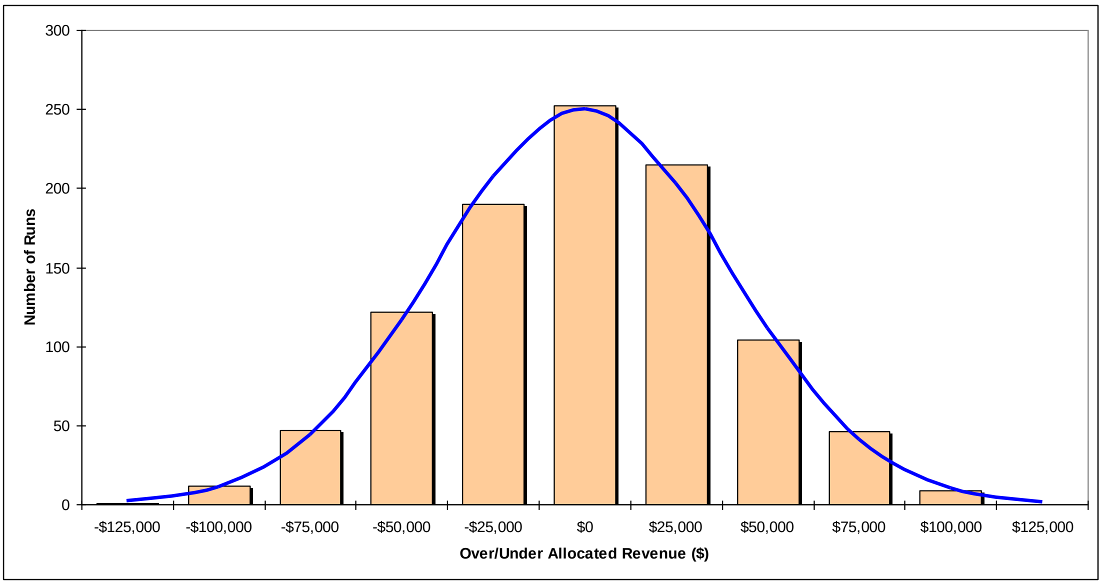
3.3 Integrated Risked Exposure to Lost Revenue

It appeared apparent in Section 3.2 that even the most compositionally stable Stream exhibits sufficient variation to warrant the full sampling OPEX. However, that was just a snapshot of the data and this section analyses the data using a more statistically structured approach.

In order to evaluate Integrated Risked Exposure to loss (as described in Section 2.7) the standard deviation of the Field E allocated revenue is required. Because of the complexity of the allocation, the standard deviation was calculated from a Monte Carlo analysis.

In each run of the Monte Carlo simulation, the weekly sample composition of Field E was varied in accordance with appropriate standard deviation figures and the allocated revenues collated. The results of such a Monte Carlo simulation for Field E are presented in Figure 14.

Figure 14 – Field E – Allocated Revenue Monte Carlo Simulation Results



In each run only the Field E composition was varied according to standard deviations typical of the Field E weekly analyses over the period June 2000 to July 2001. The histogram bars represent the number of runs in which the allocated Field E revenue lay in the bandwidth indicated on the horizontal axis – (the figures on the x-axis are the midpoints of the bands). The values on the axis refer to the difference between allocated revenue in the run compared with the average value, i.e. the over/under allocation of revenue compared with the average.

The mean allocated revenue was calculated as $2,541,000 and the associated standard deviation in the revenue figure slightly in excess of $3,000. The blue line is a plot of the normal distribution curve with the same average and standard deviation; the curve demonstrates that the allocated revenue is normally distributed.

For the example above, the risked exposure to lost revenue was calculated to be $1,270 using Equation (1). The cost of performing the Field E weekly sample is $400 per week, approximately $1,600 per month if four samples are taken. If the sampling is reduced to one week in four then the OPEX savings would be $1,200.

Collectively to the system the risk of mis-allocation is comparable with the total OPEX savings and a reduction in sampling frequency may marginally be justified in this case.

These impacts are associated with the Field E, whose throughput is relatively small and whose composition is relatively stable and hence the most likely candidate for a reduction in sampling frequency. Much larger impacts are observed for other Fields; for example the same analysis applied to Field F produced a risked exposure to lost revenue in excess of $20,000.

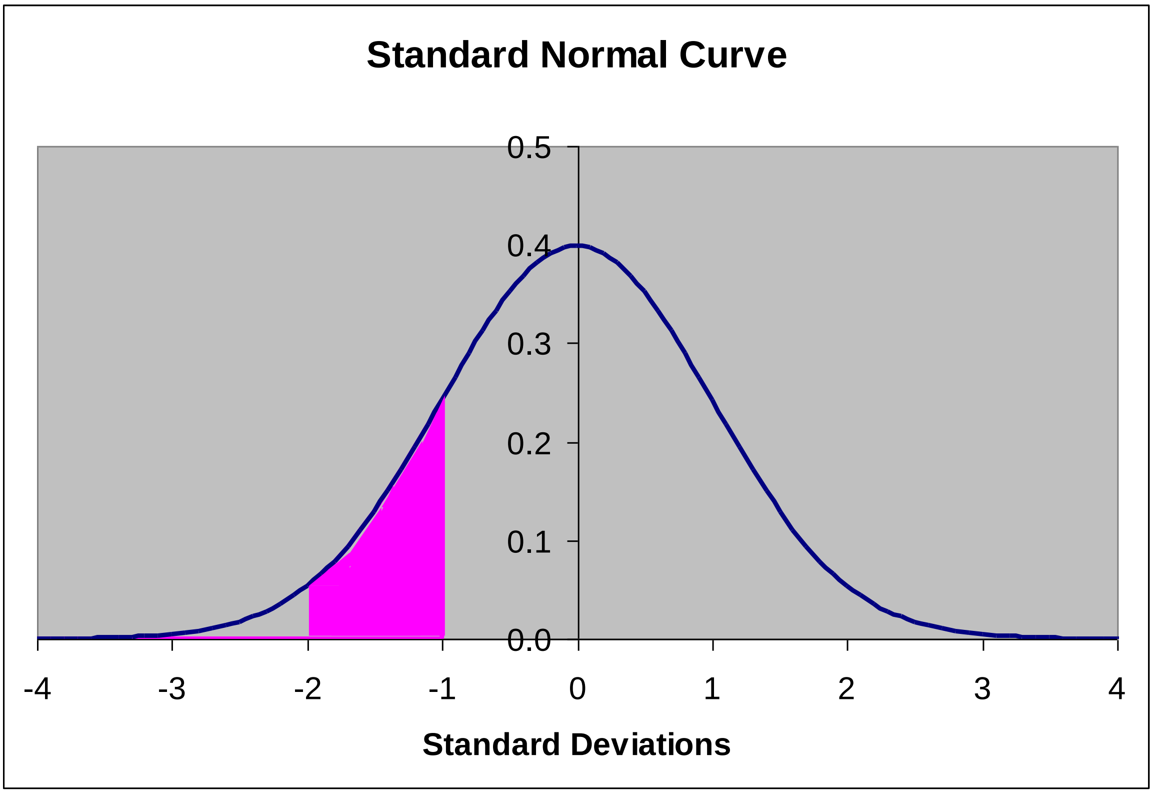
In general, there appears to be little opportunity to justify the risk of mis-allocation associated with sampling frequency reduction and these findings concur with the analyses described in Section 2.8.

4 MATHEMATICAL DERIVATIONS OF EQUATIONS PRESENTED

4.1 Risked Exposure to Lost Revenue

The allocated revenue is normally distributed and the probability is therefore described by the locus of the standard normal distribution presented in Figure 15.

Figure 15 – Standard Normal Distribution



The equation that represents the locus of the blue line is given by:

$$y = \frac{1}{\sqrt{2\pi}} e^{\left(\frac{-Z^2}{2}\right)} \qquad (2)$$

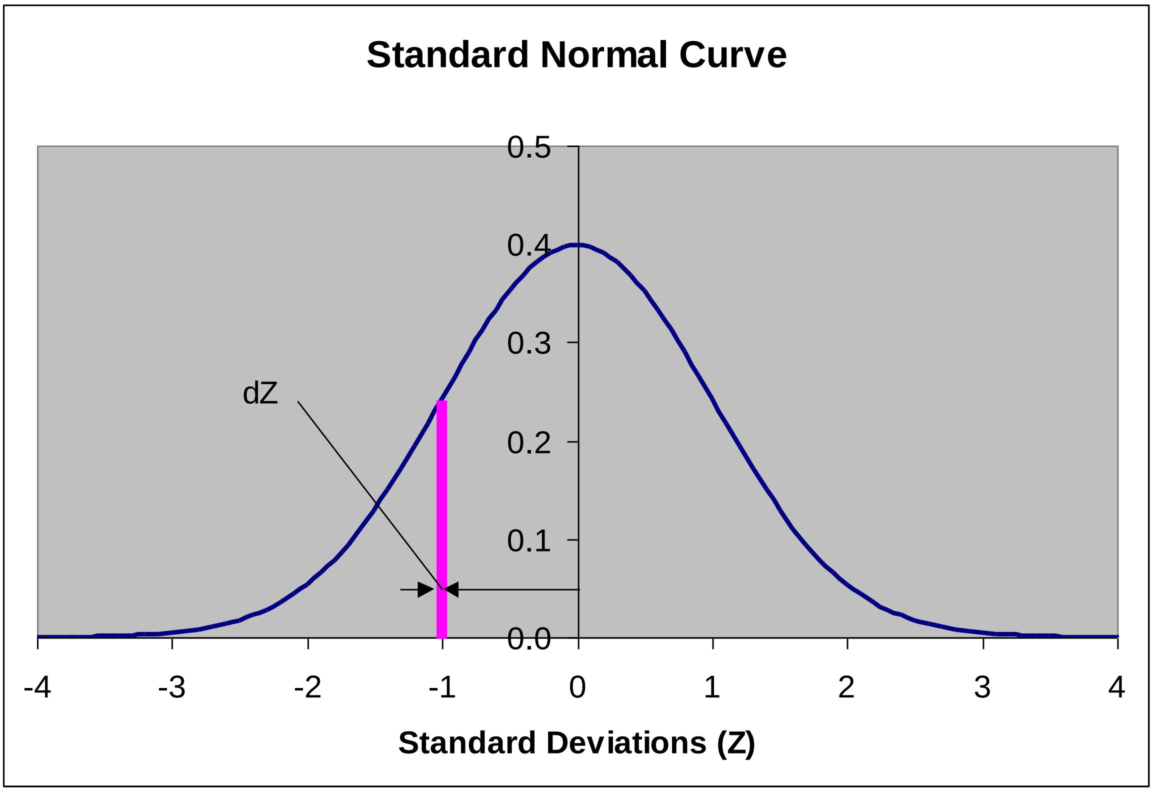
Where Z is given by:

$$Z = \frac{\left(x - x_{mean}\right)}{\sigma} \qquad (3)$$

The total area under the standard curve (blue line) is equal to 1 and the area under any section of it represents the probability that the value of X will fall between the two values of Z. In the above figure the area under the purple shaded region represents the probability that Z will lie between –1 and –2.

When perturbing input variables randomly the calculated revenue will vary about the mean value. The probability of being allocated revenue above or below the mean diminishes as the value moves from the average value. Hence in calculating the risked revenue it is necessary to multiply the revenue by the probability that that revenue value would be allocated. For example if the average revenue allocated was $100,000 and the standard deviation was $1,000 the probability of being under-allocated $1,000 (i.e. $99,000) is approximately represented by the area under the thin purple vertical line in Figure 16.

Figure 16 – Standard Normal Distribution



To calculate the risked mis-allocation exposure the under/over allocation of revenue ($x-x_{mean}$) needs to be multiplied by the area under the curve:

$$R = (x - x_{mean}) \times y \times dZ \qquad (4)$$

Where R is the risked exposure to mis-allocation of revenue and has a negative value for under-allocation (i.e. loss). Substituting from (2) into (4):

$$R = (x - x_{mean}) \times \frac{1}{\sqrt{2\pi}} e^{\left(\frac{-Z^2}{2}\right)} \times dZ \qquad (5)$$

Substituting from for ($x-x_{mean}$) from (3) and forming the integral to calculate the risked lost revenue between Z1 and Z2:

$$R = \int_{Z1}^{Z2} \left(\sigma Z \times \frac{1}{\sqrt{2\pi}} e^{\left(\frac{-Z^2}{2}\right)} \right) dZ \qquad (6)$$

This integrates to:

$$R = \left[-\sigma \times \frac{1}{\sqrt{2\pi}} e^{\left(\frac{-Z^2}{2}\right)} \right]_{Z1}^{Z2} \quad (7)$$

Hence, R is given by:

$$R = -\sigma \times \frac{1}{\sqrt{2\pi}} \left(e^{\left(\frac{-Z2^2}{2}\right)} - e^{\left(\frac{-Z1^2}{2}\right)} \right) \quad (8)$$

Expressed in terms of x:

$$R = -\sigma \times \frac{1}{\sqrt{2\pi}} \left(e^{-\left(\frac{(x2 - x_{mean})^2}{2\sigma}\right)} - e^{-\left(\frac{(x1 - x_{mean})^2}{2\sigma}\right)} \right) \quad (9)$$

The total exposure to lost revenue is found between x1 equal to minus infinity and x2 equal to x_{mean}. R then reduces to:

$$R = \frac{-\sigma}{\sqrt{2\pi}} \quad (10)$$

The value of L is the negative of R and can be expressed in terms of the uncertainty (equal to twice the standard deviation).

$$L = \frac{U}{\sqrt{8\pi}} \quad (11)$$

4.2 Expected Value of Field Allocation

The allocation to Field B is given by:

$$AL_B = \frac{M_A}{(M_A + M_B)} \times M_E \quad (12)$$

The expected quantity allocated to Field A is calculated by integrating the allocation equation with respect to its probability measure:

$$EL_B = \int_{B1}^{B2} \int_{A1}^{A2} \left(\frac{M_A}{(M_A + M_B)} \times M_E \right) * P_A * P_B dM_A dM_B \quad (13)$$

The probabilities of the range values of M_A and M_B are described by the normal distribution described by equation (2) in the Section 4.1. Substituting these in (12) produces:

$$EL_B = \int_{B1}^{B2} \int_{A1}^{A2} \left(\frac{M_A}{(M_A + M_B)} \times M_E \right) * \frac{2}{\pi} * \left(\frac{e^{\left(\frac{-2*(M_A - M_A^{ave})^2}{U_A^2}\right)}}{U_A} \frac{e^{\left(\frac{-2*(M_B - M_B^{ave})^2}{U_B^2}\right)}}{U_B} \right) dM_A dM_B \quad (14)$$

This equation has to be integrated numerically over a suitable range of meter values. The uncertainty in the export meter could also be included and the integration would be in three dimensions.

NOTATION

AL	Allocated quantity
L	Cumulative risked lost revenue
M	Metered quantity
M^{ave}	Average metered quantity
P	Probability density function
R	risked exposure to mis-allocation of revenue
U	Uncertainty in metered quantity
UR	Uncertainty in allocated revenue
x	Revenue
x_{mean}	Mean value of revenue.
y	Standard probability density function
Z	number of standard deviations from mean
σ	Standard deviation of revenue

Subscripts

$_A$	Field A
$_B$	Field B
$_E$	Export

5 REFERENCES

[1] Guide to the Expression of Uncertainty in Measurement, International Organisation for Standardisation, ISO/IEC Guide 98:1995.