

# Features of Allocation Systems Incorporating Long Pipelines

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## 1 INTRODUCTION

There are two main approaches to systems of allocation that include long pipelines. The first accounts for each user's hydrocarbons within the pipeline itself. The second method ignores the transit time in the pipeline and allocates the metered quantities exiting the pipeline based on the metered quantities input into the pipeline on the same day; using this approach parties will not be allocated precisely what they input to the pipeline on a day, but over a period of time there is an expectation that any daily gains and losses will even themselves out.

This paper examines instances when this is not necessarily true depending on the allocation equations employed. It demonstrates, using simple models and results from a real allocation system, how parties can be systematically under and over allocated hydrocarbons due to the mathematics of the allocation agreement. It goes on to examine the reasons for this unexpected and subtle bias in the allocation system and presents methods to assess the stability of the equations and approaches to eliminate allocation bias.

It also discusses the wider implications for allocation systems in general, particularly in terms of how the assumptions, equations and logic of a system should be tested at the conceptual development stage to prevent problems occurring.

In Section 2 a simple model is used to describe an allocation system associated with a pipeline. This model illustrates the basic process and presents the main features of the allocation methodology. Data from an analogous real system is presented to highlight a problem with the allocation results of such a system. In Section 3 the model is then used to analyse the allocation system behaviour without the obfuscating effects of measurement uncertainty in the real data.

## 2 PIPELINE ALLOCATION SYSTEM DESCRIPTION

A simple system incorporating a long pipeline is presented below and this is used as a basis to describe allocation issues associated with a real system.

### 2.1 Process Description

Consider two offshore platforms exporting gas to an onshore gas plant via a long pipeline such as that presented in Figure 1 below:

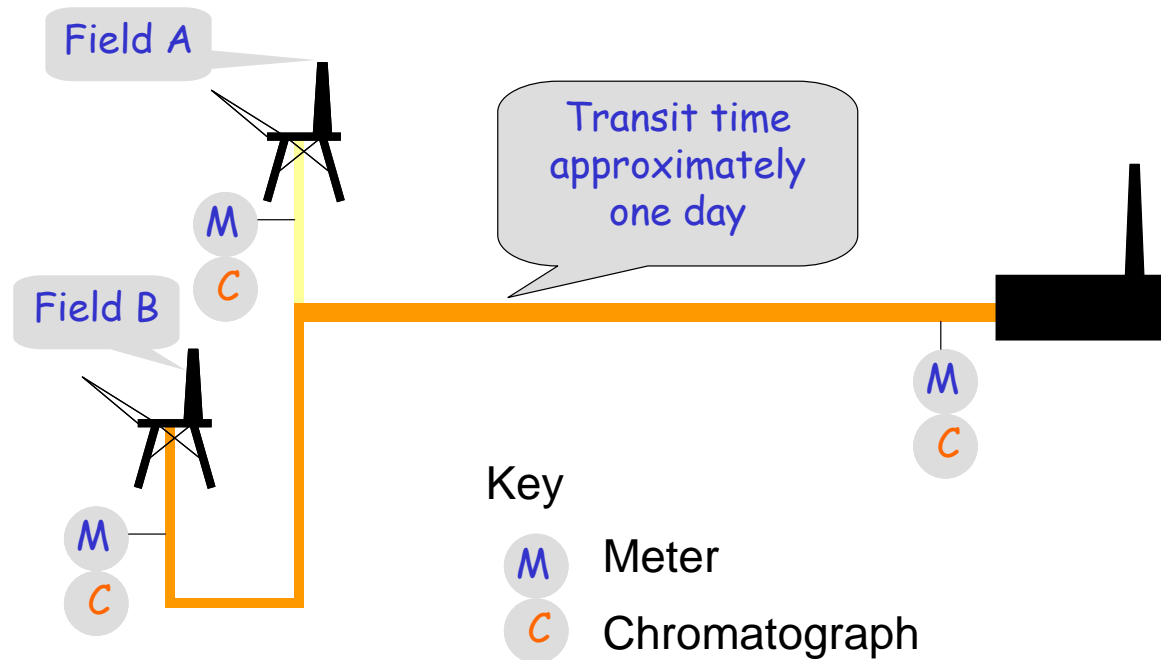


Figure 1 – Process Schematic of Two Platforms Delivering Gas to an Onshore Gas Plant via a Long Pipeline

Gas from Fields A and B is exported from two offshore platforms into a subsea pipeline where the gases are commingled and delivered to the inlet of an onshore gas plant. The two Fields' gases are of differing compositions and are continuously metered offshore and the composition determined by on-line chromatographs. Similarly the gas plant inlet flow is metered and the composition measured.

The pipeline is deemed long in that it takes approximately one day for the gas exported offshore to arrive onshore. The consequence of this is that the gas arriving onshore "today" was in fact exported "yesterday". If the composition or flow of either, or both, Field's export varies from day to day (as is almost certainly the case) then the gas metered and allocated at the inlet will have a different composition to the combined platform export on an allocation day.

## 2.2 Allocation Methodology

Typically in such systems the gas delivered to the Gas Plant is processed and exported into a transmission pipeline system and delivered to end users. Buyers of the gas nominate quantities on a day and the Fields export gas from the platforms in response to these nominations. Hence, in order to meet nominations, the quantity of gas displaced from the pipeline through the plant inlet will be roughly equal to the combined export on a day. In order for the Fields to meet their buyer's nominations, this inlet gas will normally be allocated in proportion to the Field exports. However as stated above this inlet gas will almost inevitably be of different composition to that of the combined export.

In this particular system, the gas is allocated on mass component basis and the allocation system is built on the premise of preserving a mass balance at component level. This means that whatever a Field exports at a mass component level, it should ultimately be allocated an equivalent quantity in terms of gas plant products, i.e. gas export, fuel gas, NGLs, etc. Any differences on a day are reflected in changes to the pipeline stock.

A simple scheme, and at first sight perhaps the most obvious and intuitive, is to allocate the inlet gas, at a component level, in proportion to the quantity of that component the respective Fields exported on the same day. This is illustrated with a simple numerical example in Table 1:

**Table 1 Metered and Allocated Quantities**

	Field A	Field B	Inlet
Metered	10	20	33
Allocated	11	22	33
Stock Change	-1	-2	

These figures could pertain to any component and be on a mass or molar basis – since mass and molar quantities are conserved across the pipeline. For illustrative purposes it will be deemed that the above quantities refer to C1 (methane) and are in mass units.

Since it takes roughly one day for the gas to pass through the pipeline the gas being allocated at the gas plant inlet is effectively the previous day's export gas but it is being allocated based on today's export gas quantities. Slightly more mass of C1 is metered at the inlet than is exported offshore on that day and this is simply due to the fact that there was a different commingled composition exported the previous day.

In order to preserve the component mass balance the difference between what a Field exported and what it was allocated at the inlet is accounted for by a change in the pipeline stock allocated to that Field. The pipeline stock change is calculated as the difference between the allocated inlet and offshore export quantities. The stock change for both Fields in the example is negative indicating more of that component has been removed from the pipeline than has been exported into it that day. The next day the figures could be positive varying in accordance with the fluctuations in offshore export flows and compositions.

Each Field has a total pipeline stock which represents the cumulative difference between what it has exported and what it has been allocated at the inlet. The total pipeline stock, i.e. the sum of the two Fields, represents the physical contents of the pipeline. The daily stock change is added or subtracted from each Field's previous day closing balance to obtain a new closing balance of pipeline stock.

The above is expressed in equation format below. The allocated inlet to Field A is given by:

$$AI_{A,c} = MI \times xI_c \left[ \frac{ME_A \times (xE_{A,c})}{ME_A \times (xE_{A,c}) + ME_B \times (xE_{B,c})} \right] \quad (1)$$

The change in stock by:

$$\Delta S_{A,c} = ME_A \times (xE_{A,c}) - AI_{A,c} \quad (2)$$

And the new closing stock by:

$$SC_{A,c} = SO_{A,c} + \Delta S_{A,c} \quad (3)$$

The closing stock becomes the opening stock for the next day. Similar equations can be written for Field B.

Though a Field may be allocated more or less of a component at the inlet than it exported on a day, reflected in a positive or negative stock change, it might be expected that these gains and losses would even themselves out over a period of time. The assumption that "you get out what you put in" over a period of time appears intuitive and at first sight reasonable.

The above equations are simple but it should be noted that they are non-linear. Also the fact that the length of the pipeline introduces a time factor implicitly into the calculations means that the equations are also dynamic in nature. Such systems of simple non-linear, dynamic equations have been found to behave unexpectedly and exhibit instability. In fact there is now a whole field of mathematics and engineering concerned with the nature of stability and chaos

associated with such equations [1]. With this in mind it is perhaps not surprising that unexpected results may follow from the allocation equations presented above.

### 2.3 Allocated Pipeline Stock Problem

Table 2 presents allocated pipeline stock data from a real allocation system\* that operates in a similar manner to the simple system described in Sections 2.1 and 2.2.

**Table 2 Real Pipeline Stock**

	<b>Field Alpha</b>	<b>Field Bravo</b>	<b>Total</b>
	<b>kg</b>	<b>Kg</b>	<b>kg</b>
<b>N2</b>	166,786	255,147	421,933
<b>CO2</b>	(-504,728)	4,816,148	4,311,420
<b>C1</b>	5,206,866	14,366,841	19,573,707
<b>C2</b>	3,977,723	2,397,806	6,375,529
<b>C3</b>	2,785,383	(-812,473)	1,972,910
<b>IC4</b>	751,988	202,924	954,912
<b>NC4</b>	1,552,043	102,636	1,654,679
<b>IC5</b>	328,169	412,012	740,181
<b>NC5</b>	410,196	57,509	467,705
<b>C6</b>	(-849,384)	236,605	(-612,779)
<b>C7</b>	(-163,564)	296,035	132,471
<b>C8+</b>	354,973	300,008	654,981
<b>Total</b>	14,016,451	22,631,198	36,647,649

There are some negative values (in brackets) which means, for example, that Field Alpha has been allocated more CO<sub>2</sub> at the gas plant inlet than it has exported. The problem with these figures is illustrated more explicitly when expressed on a mass percentage basis as shown in Table 3, especially when compared against the typical Field export compositions presented in Table 4.

**Table 3 Real Pipeline Stock Compositions**

	<b>Field Alpha</b>	<b>Field Bravo</b>	<b>Total</b>
	<b>wt%</b>	<b>wt%</b>	<b>Wt%</b>
<b>N2</b>	1.2%	1.1%	<b>1.2%</b>
<b>CO2</b>	(-3.6%)	21.3%	<b>11.8%</b>
<b>C1</b>	37.1%	63.5%	<b>53.4%</b>
<b>C2</b>	28.4%	10.6%	17.4%
<b>C3</b>	19.9%	(-3.6%)	5.4%
<b>IC4</b>	5.4%	0.9%	2.6%
<b>NC4</b>	11.1%	0.5%	<b>4.5%</b>
<b>IC5</b>	2.3%	1.8%	2.0%
<b>NC5</b>	2.9%	0.3%	<b>1.3%</b>
<b>C6</b>	(-6.1%)	1.0%	(-1.7%)
<b>C7</b>	(-1.2%)	1.3%	0.4%
<b>C8+</b>	2.5%	1.3%	1.8%
<b>Total</b>	100.0%	100.0%	100.0%

\* Though the values presented are taken from a real system, the Field names are fictitious in order to anonymise the data.

**Table 4 Average Export Compositions**

	<b>Field Alpha</b>	<b>Field Bravo</b>
	wt%	wt%
<b>N2</b>	1.8%	0.5%
<b>CO2</b>	9.9%	12.8%
<b>C1</b>	48.7%	61.7%
<b>C2</b>	14.6%	11.6%
<b>C3</b>	14.4%	7.2%
<b>IC4</b>	1.9%	1.1%
<b>NC4</b>	5.0%	2.4%
<b>IC5</b>	1.1%	0.7%
<b>NC5</b>	1.3%	0.8%
<b>C6</b>	0.8%	0.8%
<b>C7</b>	0.2%	0.4%
<b>C8+</b>	0.1%	0.1%
<b>Total</b>	100.0%	100.0%

In the absence of measurement uncertainty it might be expected that the Fields' pipeline stock compositions would be similar to their export compositions - this is clearly not the case. As this is real data, the drifts in pipeline stock compositions may be due to the effect of small systematic biases in the measurements\* accumulated over a period of time.

However, the total pipeline stock composition shown in Table 3 seems more reasonable than the individual Field compositions, having only one negative value and appearing more consistent with the combined Fields' export composition. The emboldened figures indicate values that lie between the Field export composition values. If measurement error was the sole cause of the anomalous stock compositions it would be expected that the total stock composition would be as unrepresentative as the Field values.

The only other cause of these anomalous stock compositions is the allocation system methodology itself and in particular the validity of the assumption presented at the end of Section 2.2, i.e. that gains and losses cancel over a period of time.

In the next section the validity of this statement is examined using the simplified model, described in Sections 2.1 and 2.2, to eliminate the effects of measurement uncertainty.

### **3 ANALYSIS OF ALLOCATION SYSTEM EQUATIONS**

#### **3.1 Simplified Pipeline Allocation Model**

In order to determine if the allocation equations themselves are contributing to the anomalous stock values a simplified spreadsheet model was constructed. This was required so that the effect of measurement uncertainty could be eliminated. The model is illustrated schematically in Figure 2.

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\* In using the term measurement bias in this context it should be emphasised that it is not being suggested that there were any faults with the meters or chromatographs. The instruments were of a fiscal standard, calibrated correctly and properly maintained. However, any real instrument is not perfect and will exhibit small biases which will accumulate over a period of time. The allocation system stock changes are small compared with the metered quantities used to calculate them and as the stock change is calculated as the difference between these relatively large flow rate figures, even small biases appear amplified in the pipeline stock values.

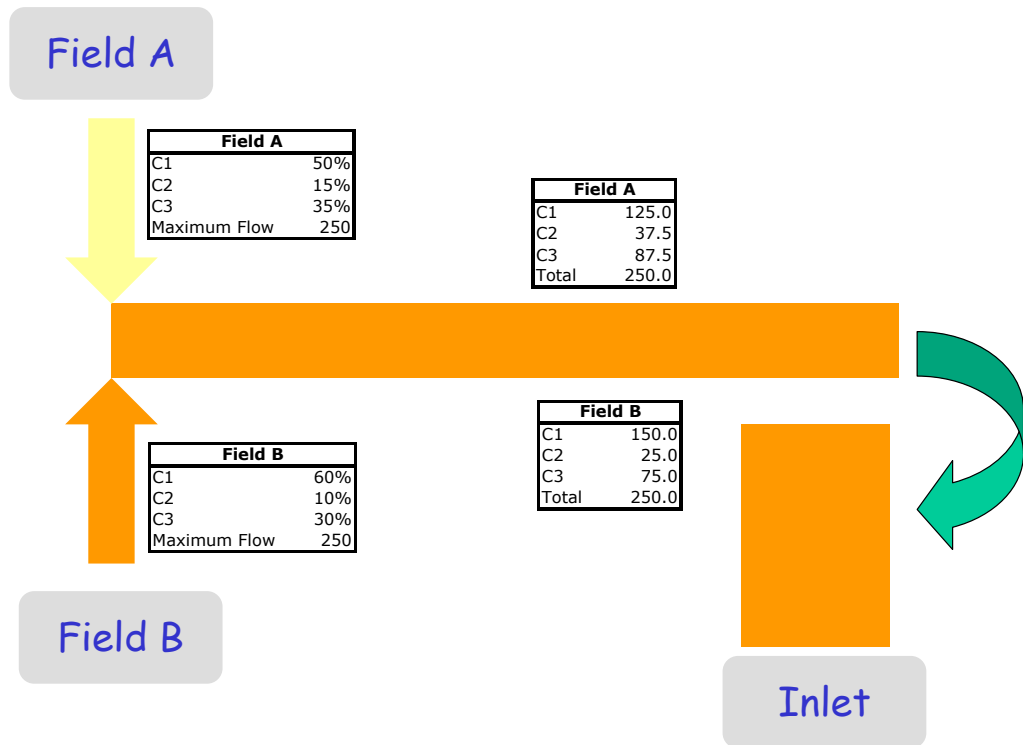


Figure 2 – Schematic of Simplified Pipeline Allocation Model

The model consists of two Fields, A and B, exporting at the same entry point into a pipeline. The gas exported has three components (C1, C2 and C3) and the flow is incompressible so that the total exported offshore is reflected by an equal quantity being displaced from the pipeline and metered at the inlet onshore. The pipeline is such that it has a minimum residence time of one day. The Fields have different compositions but these remain fixed. The flow rates of the two Fields are allowed to vary independently and randomly between zero and a maximum value on a daily basis. The units are arbitrary but may be considered to be mass or molar. The initial pipeline stock attributed to each Field is equal to one day's maximum flow. The compositions, flows and initial stocks are indicated on Figure 2.

Though the total metered inlet exactly sums to the total Field export on a day the compositions will generally be different as the Field flows are varying independently. The model was run using a Monte Carlo approach to vary the Field export flows and run over 100 days. The allocation system described in Section 2.2 was applied in the model and the daily pipeline stocks calculated for each Field at a component level. The results are presented in Figure 3 for Field A and Figure 4 for Field B.

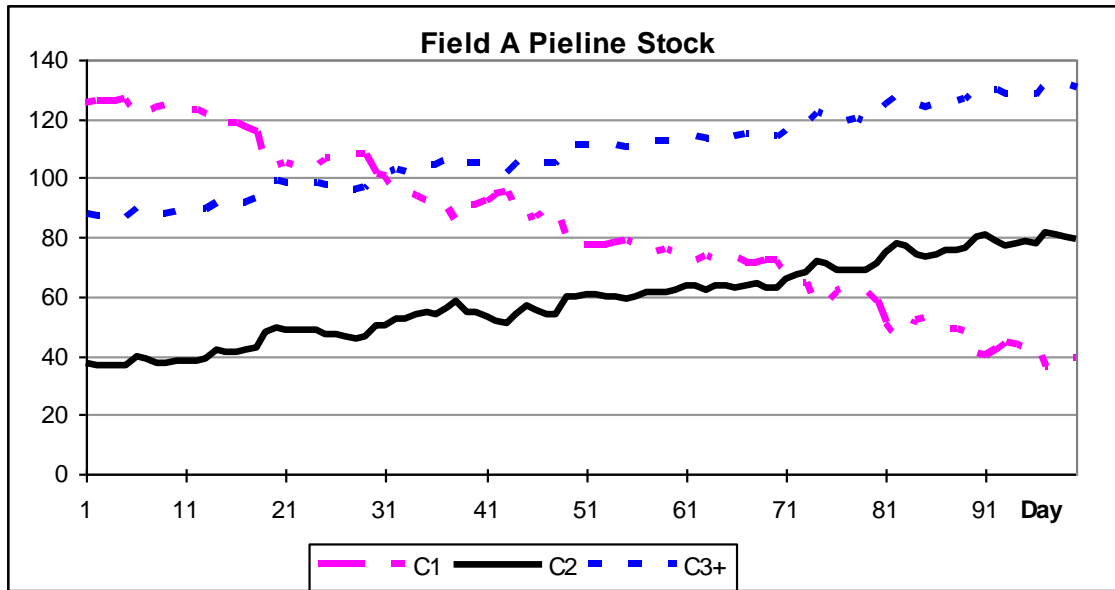


Figure 3 – Field A Pipeline Stock

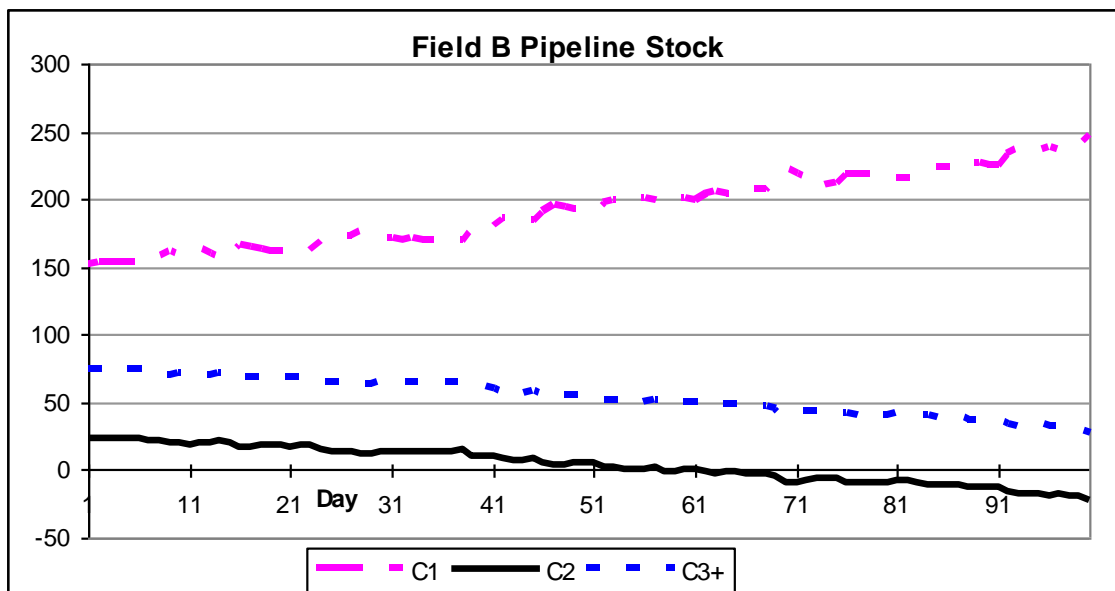


Figure 4 – Field B Pipeline Stock

The charts clearly show that the component stock compositions of the two Fields systematically drift. Field A's C1 stock drops whilst Field B's rises by an equal and opposite quantity. The converse behaviour is observed with C2 and C3. These component stock levels continually diverge, either rising indefinitely or falling indefinitely below zero.

Since in this theoretical model the measured flows and compositions are perfect it would be expected that the total stock composition does not exhibit systematic component drift and this is confirmed in Figure 5.

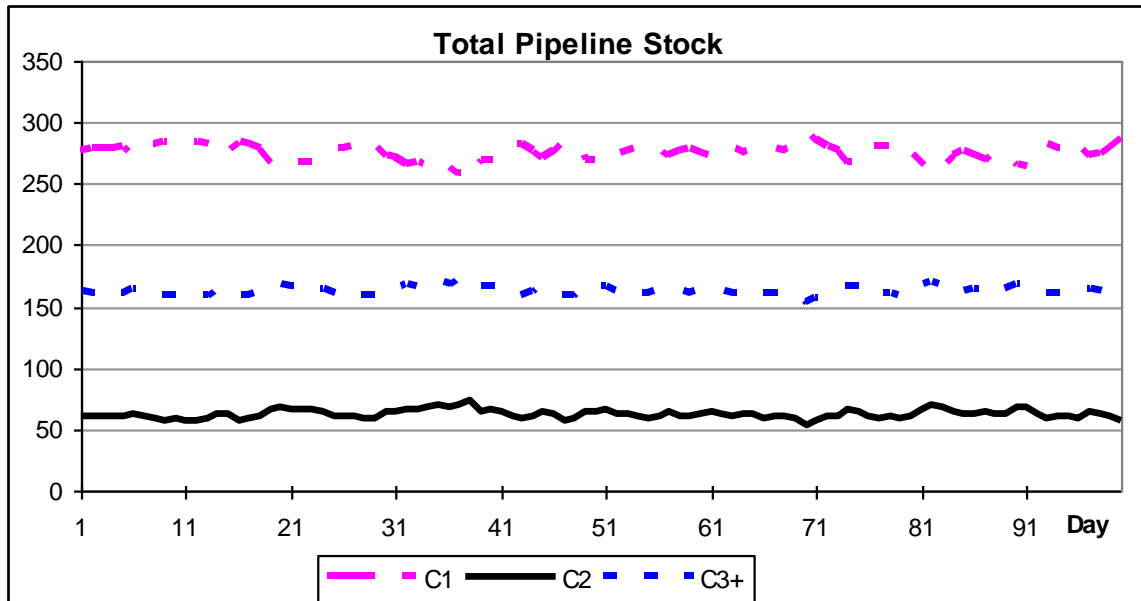


Figure 5 – Total Pipeline Stock

### 3.2 Conclusions from Allocation Model Results

The following conclusions maybe drawn from these results (and further analysis conducted with the model not presented in the paper):

- At a component level, the expectation that the gains and losses cancel over a period of time is not correct
- The difference between export and allocated inlet diverges with time at a component level
- The more dissimilar the Field compositions the faster the divergence
- The Field richer in a component is systematically under-allocated that component at the inlet – i.e. its pipeline stock of that component increases
- A Field's total pipeline stock remains roughly constant meaning that decreases in components are offset by increases in other components
- The Field level compositional stock drift is solely due to the allocation methodology
- The allocation equations are unstable

The conclusion that a Field richer in a component will experience a systematic increase in pipeline stock of that component and vice versa for the other Field, is observed in the real data presented in Table 3 and Table 4 for nine out of the twelve components. This indicates that the drift in the real stocks is partly due to the allocation system equations and not solely measurement uncertainty effects.

At this point it may be observed that though these results are interesting the fact that the under allocation of components that a Field experiences are offset by gains in other components, it may be asserted that the overall material impact of this is unimportant. However, the component that a Field is compensated with may be less valuable than the one it has been under allocated, e.g. compensation with CO<sub>2</sub> or N<sub>2</sub> at the onshore inlet for hydrocarbons is a loss of value. Similarly relatively valuable NGL components compensated for by gas components (under current market conditions) represents a loss also.

The model indicates that in a real system Fields rich in NGLs will tend to be systematically under-allocated these components at the inlet. Similarly Fields rich in CO<sub>2</sub> will be under-allocated this inert at the inlet at the expense of the other Field. It appears there will be winners and losers as a consequence of the allocation equations used, the system is not equitable.



In fact this is clearly illustrated in the real data presented in Table 2. Field Alpha has lower CO<sub>2</sub> export content than Bravo; it also has a negative CO<sub>2</sub> stock meaning it has been allocated more CO<sub>2</sub> at the inlet than it has exported. Also, Field Alpha is richer in the NGL components and experiences an increase in stock of virtually all these heavier components meaning it has been under allocated these components at the inlet. Hence, the data suggests that Alpha has been significantly disadvantaged in that it has been systematically allocated more CO<sub>2</sub> and less NGLs at the plant inlet than it exported.

Some allocation systems may not allocate pipeline stock operating on the assumption that “you get out what you put in” over a period of time and hence allocation of stocks is not required. Whether they are explicitly calculated or not they do represent a record of the difference between what has been put into and taken out of the pipeline by a Field. Just because pipeline stocks are not calculated doesn't mean that there isn't a problem with the allocation system.

The expectation that the gains and losses in pipeline stock would tend to cancel over a period is palpably untrue. The next section examines in detail why this is the case.

### 3.3 Analysis of Stock Drift

Using the simplified model described in Section 3.1, and varying the Field export flow rates in a systematic fashion (rather than randomly), the details of the allocation calculations can be analysed. Consider the case where initially both Fields are producing at maximum flow under steady state conditions. This is presented schematically in Figure 6:

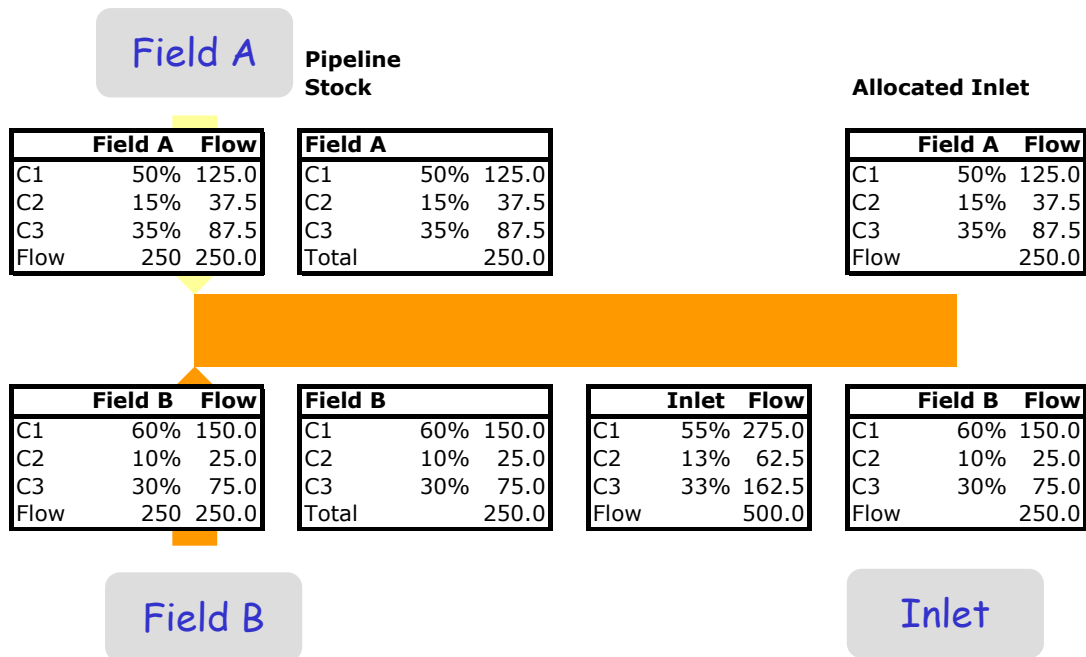


Figure 6 – Day 1 Steady State

As can be seen under such conditions the metered inlet exactly equals the sum of the exports at a component level. Under these circumstances both Fields' allocated inlet is exactly what they exported and their stock remains unchanged. On Day 2 Field A shuts down whilst Field B's production remains steady:

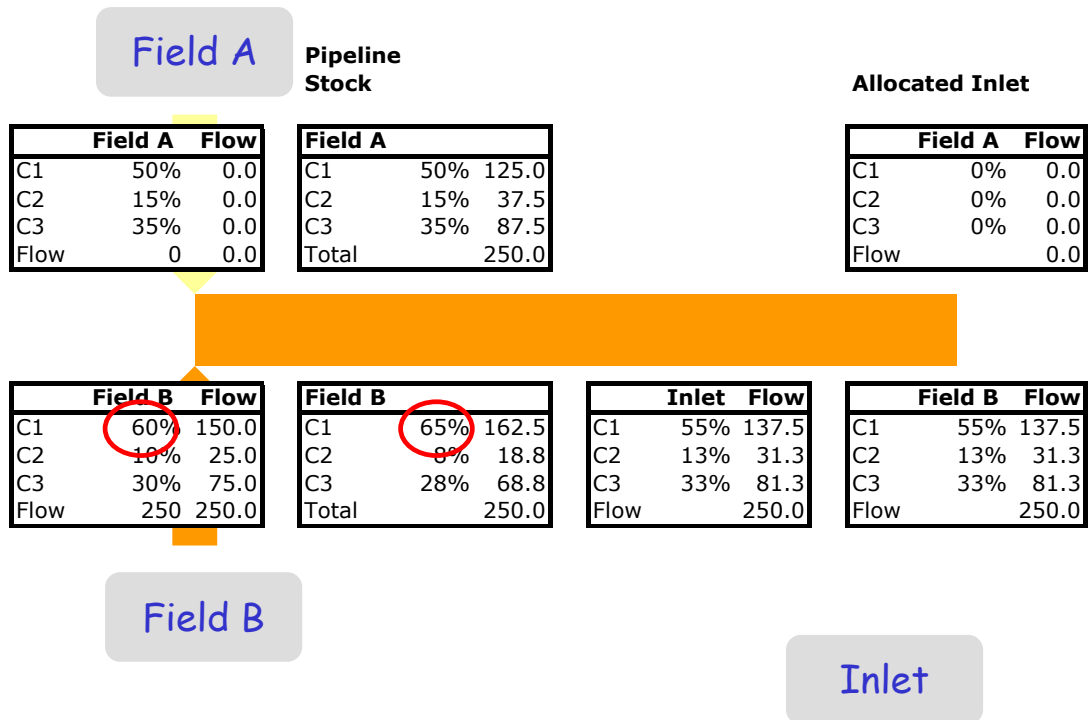


Figure 7 – Day 2 Field A Shuts Down

With only one Field flowing exactly half the pipe contents are displaced through the inlet which comprises a 50:50 mixture of Fields A and B, all of which is allocated to Field B on Day 2. Analysing C1 specifically: since Field B is richer in this component than Field A it is allocated 12.5 units less at the inlet than it exported and its stock increases by this amount. Conversely, it experiences a drop in C2 and C3 stock. It can also be seen that Field B's C1 stock expressed as a percentage is now higher than its export figure (highlighted by ellipses) – it has started to diverge.

Field A's stock is constant since it wasn't flowing; however the next Day Field A starts up again:

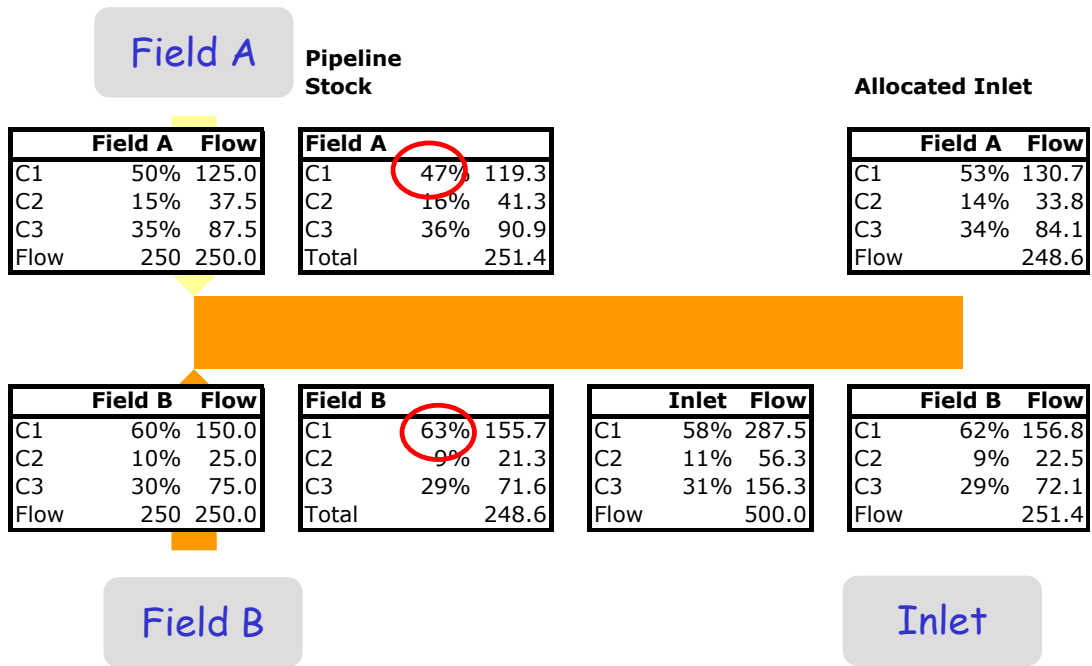


Figure 8 – Day 3 Field A Starts Up Again

Now with both Fields flowing the entire pipeline contents are displaced through the inlet and this now comprises a 25:75 mixture of Fields A and B respectively. Being rich in Field B gas, the C1 metered at the inlet is greater than the combined C1 export and both Fields are allocated more C1 at the inlet than they exported and hence their C1 pipeline stocks drop. In order for Field B's C1 stock to return to its initial Day 1 level it needed to be allocated 12.5 more units at the inlet than it exported but it is only allocated 6.8 units more – hence its C1 stock drops but not back to the initial level.

Though Field B has partially returned its C1 stock towards its initial value, Field A's C1 stock has now dropped below its initial level and expressed as a percentage is now lower than its export figure (highlighted by ellipses) – it too has started to diverge in the opposite direction to Field B. In fact the more Field B returns its C1 back to its initial level the more Field A's C1 stock is caused to drop. It appears that any deviation in one Field's stock can not be corrected without causing the other Fields stock to diverge in the opposite direction.

If the system remained in steady state the stocks would remain unchanged. However, to continue the analysis Field B now shuts down:

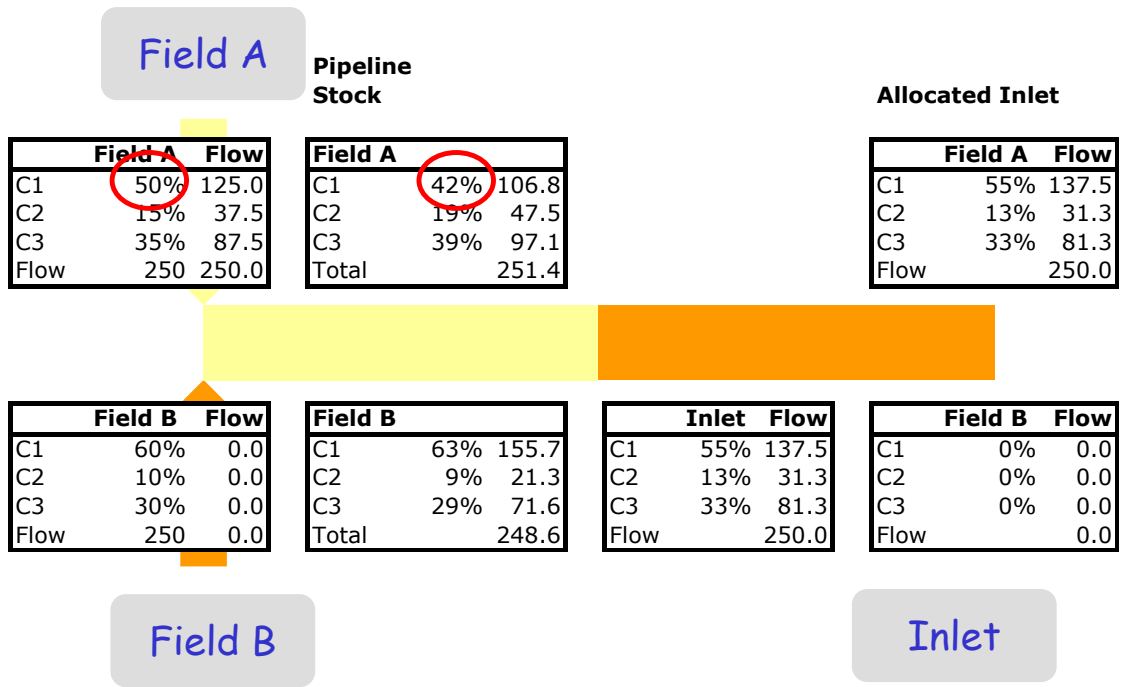


Figure 9 – Day 4 Field B Shuts Down

Again with only one Field flowing exactly half the pipe contents are displaced through the inlet and this is a 50:50 mixture of Fields A and B, all of which is allocated to Field A. Now Field A is allocated 12.5 units more of C1 at the inlet than it exported and its stock consequently decreases by this amount causing it to diverge further. Now Field B restarts:

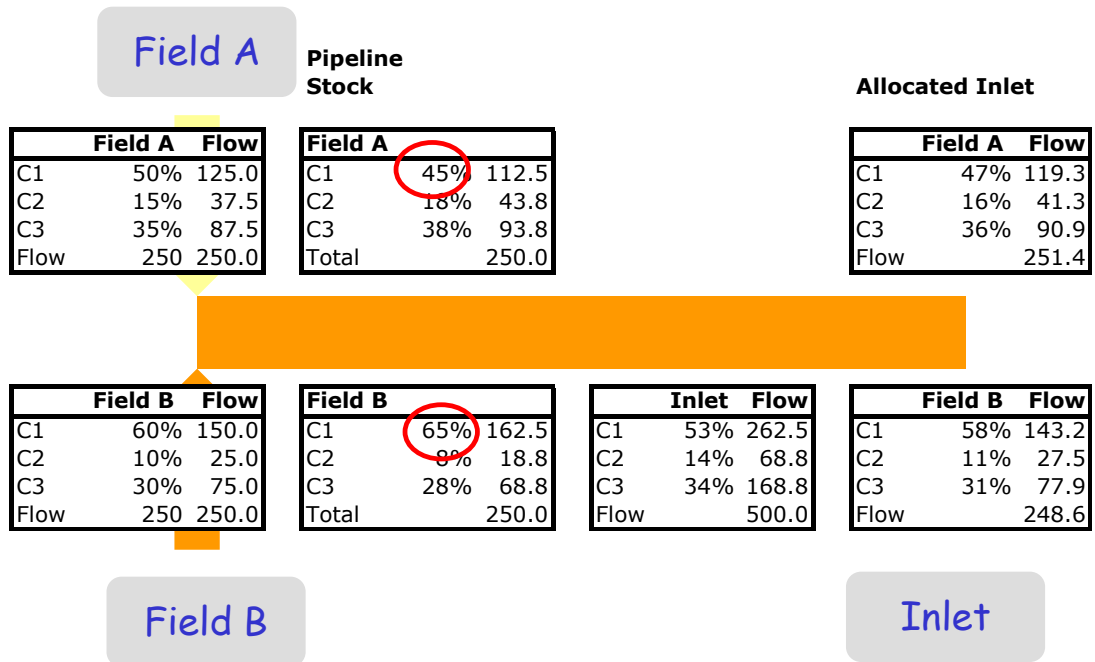


Figure 10 – Day 5 Field B Starts Up Again

The inlet is 75:25 mixture of Fields A and B respectively. The inlet being Rich in Field A results in both Fields being allocated less C1 at the inlet than they exported and hence their C1 pipeline stocks increase. Field A's C1 stock is corrected to some extent but at the expense of Field B's C1 stock diverging again.

After 5 days, Field A's C1 stock is lower than its export content and Field B's is higher than its content, they are diverging in opposite directions. As observed in the Monte Carlo simulations the Field richer in a component experiences a systematic increase in its stock of that component and the Field leaner in that component experiences an equal and opposite decrease.

Having observed the effect and analysed why it occurs the next section presents modified equations to eliminate the instability in the equations.

## 4 ALTERNATIVE ALLOCATION METHODOLOGY

### 4.1 Stock Composition Based Allocation Model

Instead of using a Field's gas export composition to allocate the inlet, the modified approach is to use its stock composition. Equation (1) is amended to:

$$AI_{A,c} = MI \times xI_c \left[ \frac{ME_A \times (xSO_{A,c})}{ME_A \times (xSO_{A,c}) + ME_B \times (xSO_{B,c})} \right] \quad (4)$$

Where,

$$xSO_{A,c} = \frac{SO_{A,c}}{\sum_c SO_{A,c}} \quad (5)$$

Equations (2) and (3) are unchanged.

This minor change in the mathematical approach generates similar allocation results to the export composition based approach. However, though similar there are small differences in the daily allocated quantities. Because the stock compositions more closely reflect the measured inlet composition, and there is a feedback mechanism in that the Fields' stocks are influencing the allocated quantities at the inlet, the stocks remain stable. This is illustrated using the simplified model but with the allocation amended in accordance with equation (4) - the results are presented in Figure 11 and Figure 12.

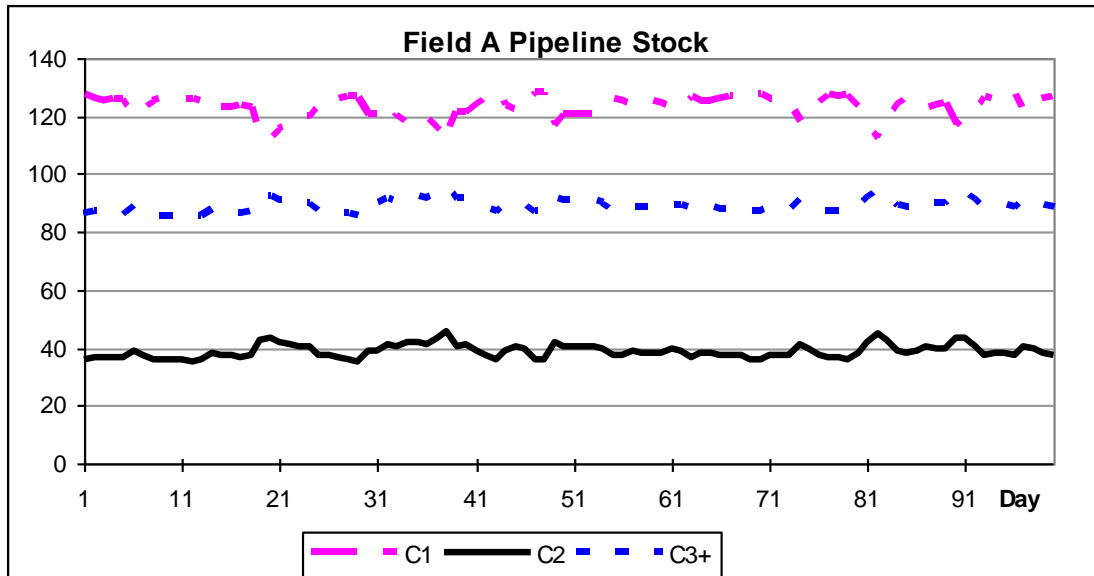


Figure 11 – Field A Pipeline Stock – Stock Based Allocation

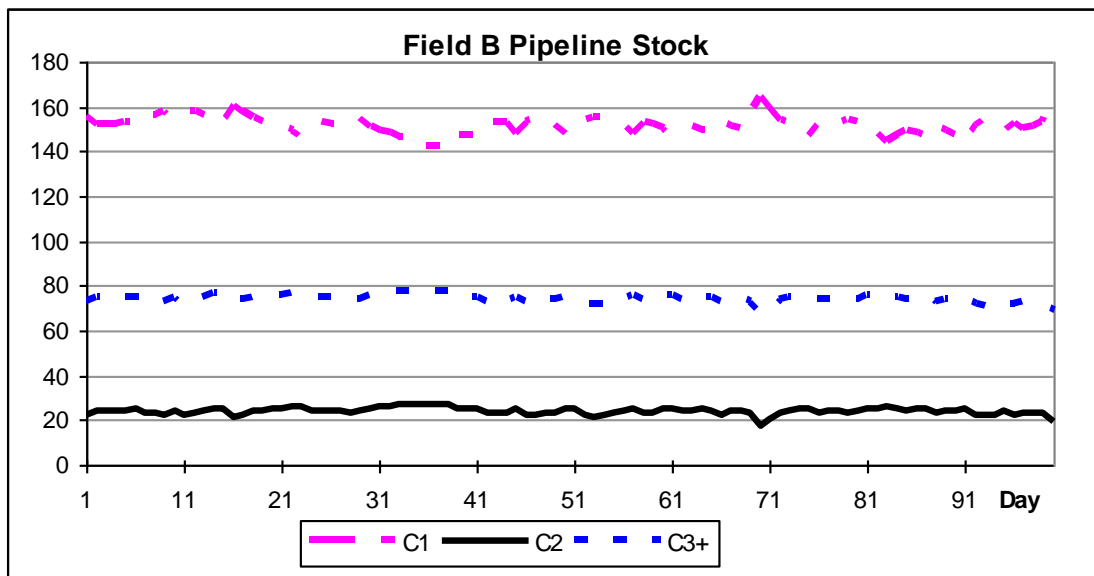


Figure 12 – Field B Pipeline Stock – Stock Based Allocation

Returning to Day 3 of the example presented in Section 3.3 and modifying the allocation calculations in accordance with equation (4):

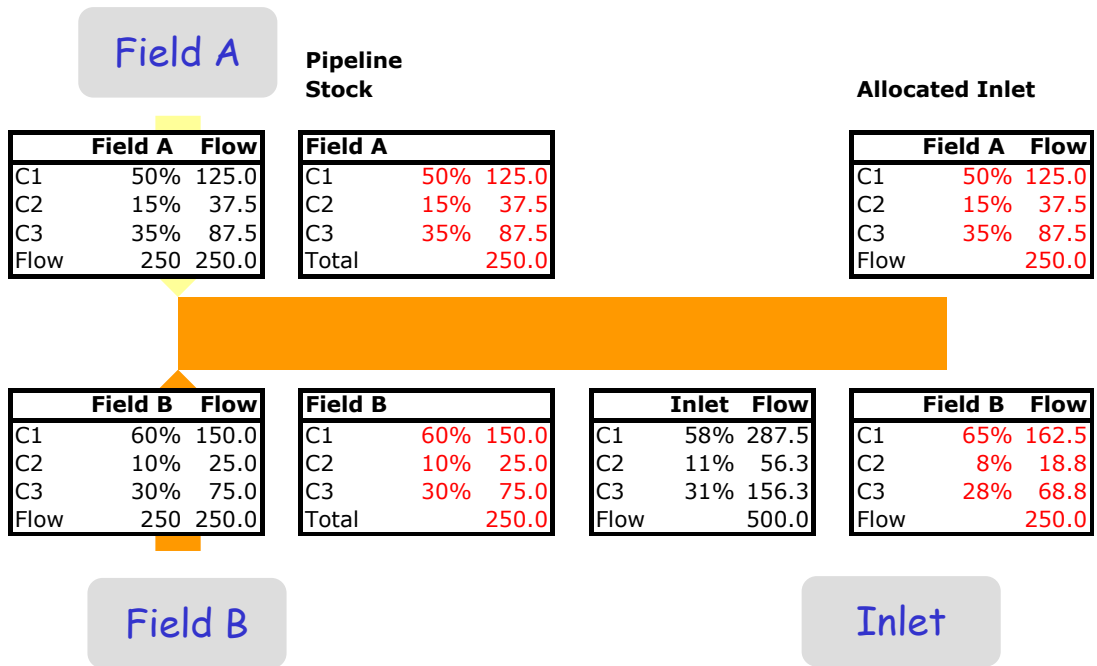


Figure 13 – Day 3 Field A Starts Up Again – Stock Based Allocation

Now the closing stock compositions are exactly equal to the Field export compositions. Field B's opening stock C1 content was 65% which is higher than its export composition but this means that in the stock based allocation Field B is allocated a greater proportion of the inlet than in the export based allocation and accordingly results in a greater reduction in its pipeline stock.

Since the stock content of a component itself is involved in the calculation there is a feedback mechanism so that the allocation equations now tend to rectify the stock composition back towards the export composition. For example, if the stock content of a component rises, on the next day it automatically influences the allocation such that the Field is allocated more of that component and hence tends to reduce its stock level of that component.

There is one word of caution with this approach however. Should any of a Field's component stock levels become negative the allocation results from then on become wildly unstable. This problem is easily avoided however by allocating zero of that component in the inlet to that Field and any export of that component is added to its stock.

#### 4.2 Application to Real Data

Finally could the modified allocation approach have helped in the real system described in Section 2.3?

Figure 14 shows the pipeline stock propane content allocated to the two Fields over a period of approximately 3 months using the existing export composition based approach. As can be seen Field A's stock is higher than its export C3 content and Field B has a negative C3 content; neither stock level reflects the actual export C3 content of the Fields.

Using the same measurement data for the period, the allocation was adjusted to be stock composition based and the results are presented in Figure 15. As can be seen the stock compositions are in good agreement with the actual export compositions throughout the period.

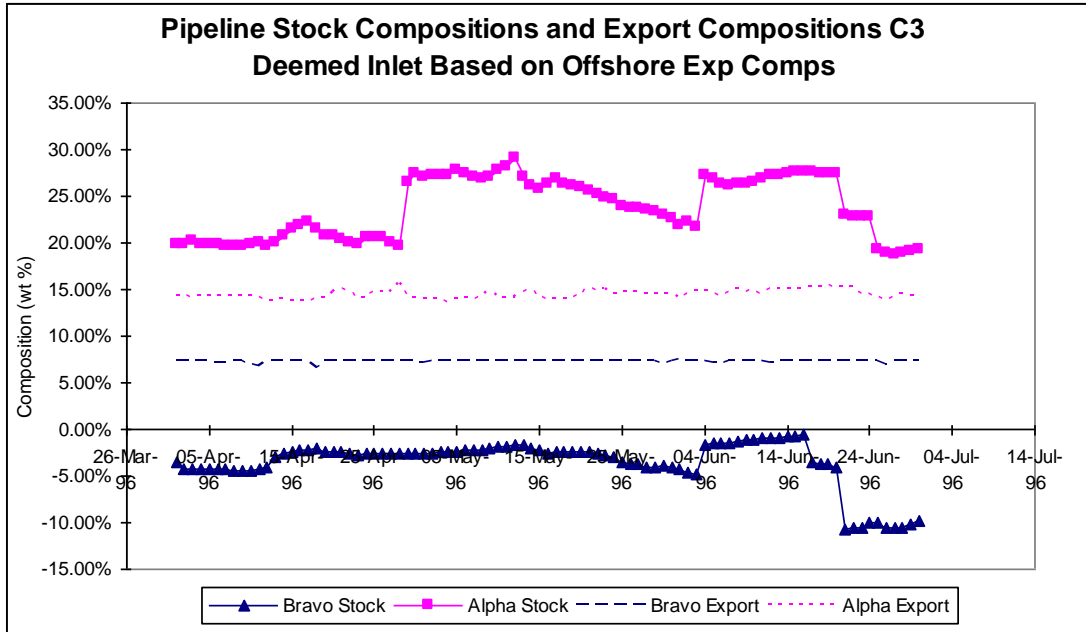


Figure 14 – Real System – Allocated Pipeline Stock for Propane (C3)

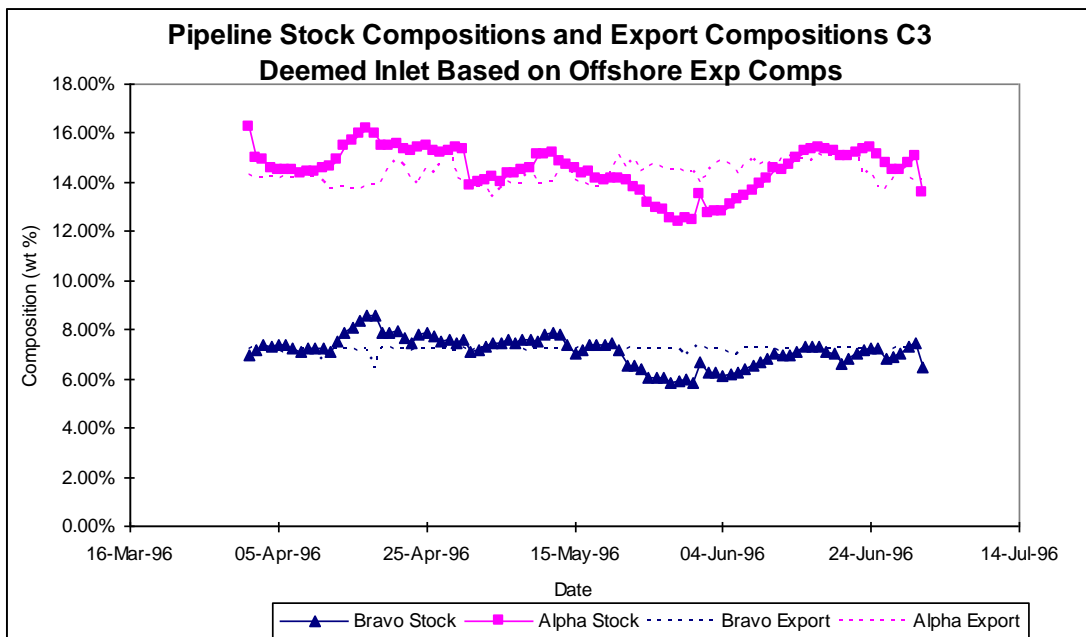


Figure 15 – Real System – Stock Composition Based Allocated Pipeline Stock for Propane (C3)

## 5 CONCLUSIONS

Allocation equations associated with pipeline systems will generally be non-linear and include a dynamic element. Such equations are well recognised to be susceptible to unstable behaviour [1].

The real system described in Section 2 had been operating for a number of years before the problem with the allocation methodology was detected. The reason for this is that the effect is subtle and obscured by measurement uncertainties. In addition, the expectation that “you get out what you put in” over a period of time appears intuitive and at first sight reasonable but can be demonstrated to be untrue.



There exists a preference for simple transparent allocation systems. However, as has been demonstrated in this paper, simple systems can produce unexpected and inequitable results. Though the paper has been centred on pipeline allocation the issues have wider implications for allocation systems in general and in particular the methods used to establish their robustness, stability and equitability.

The following points may be concluded from the analysis carried out in this paper:

- Even simple equations can produce unexpected results and lead to bias in allocation
- The requirement to test rigorously allocation system assumptions, equations, methods and logic is necessary at the design stage
- The use of simplified models such as those presented in Section 3.1 are powerful tools in testing understanding the robustness and equitability of allocation systems
- Allocation systems can be designed that are stable, robust and equitable.

## NOTATION

AI	Allocated Inlet
ME	Metered Export
MI	Metered Inlet
xE	Mass fraction of a component measured at Export
xl	Mass fraction of component c measured at Inlet
xSO	Mass fraction of component c in allocated Pipeline Stock
CS	Closing Pipeline Stock
OS	Opening Pipeline Stock
$\Delta S$	Change in pipeline stock

## Subscripts

A	Field A
B	Field B
c	component

## 6 REFERENCES

- [1] Non-Linear Dynamics and Chaos, Steven H. Strogatz, published 2000, ISBN 0-7382-0453-6.