

# Experiences in the Use of Uncertainty Based Allocation in a North Sea Offshore Oil Allocation System

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## 1 INTRODUCTION

This paper describes the development, application and experience in the use of uncertainty based allocation in ExxonMobil's Beryl Area offshore oil allocation system.

The metered products from a common production facility, in which hydrocarbons from different fields are commingled, are usually allocated based on the estimated production from those fields. This may be in proportion to the estimated production or by-difference if one estimate is much better than the other. An alternative approach, termed "Uncertainty Based Allocation" (UBA), takes into account the uncertainty in the estimated production from each field. It is particularly appropriate where the uncertainty in estimated production from the fields is significantly different.

In the example presented, stabilised oil produced from the Beryl platform is allocated between two fields: Skene and Buckland. The Skene Field's production is based on a metered quantity whereas the Buckland Field's is estimated from well tests. The confidence in the estimated production from the metered field is much better than that of the well tested field. Therefore the application of UBA, where more "weight" is given to the metered field's estimated production, was deemed appropriate. The UBA was implemented in the live allocation system at the end of 2001 and has been used since that date.

This paper describes historical development of the UBA methodology for this particular application and presents actual allocation data covering the complete allocation period. It also compares the UBA results obtained with those which would have been obtained with proportional allocation and by-difference allocation.

The paper is structured as follows:

- Section 2 describes the platform topsides process
- Section 3 describes the historical development of the uncertainty based allocation system for the Beryl platform and those considerations which influenced its development
- Section 4 presents the results of the actual allocation over a 5.5 year period and also illustrates what would have been allocation if the system had been a by-difference or proportional system
- Section 5 provides some brief conclusions
- Section 7 presents the mathematical derivation of the uncertainty based allocation equation.

## 2 TOPSIDES PROCESS DESCRIPTION

### 2.1 Processing Configuration

An overview of the Beryl Alpha topsides process is presented schematically in Fig 1.

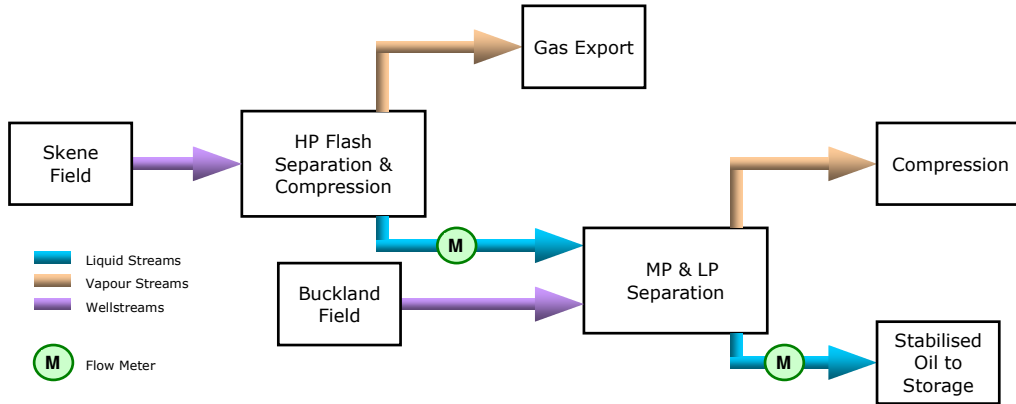


Fig. 1 – Schematic Overview of Topsides Process

Skene Field fluids are produced through a subsea pipeline and flashed in a high pressure (HP) Separator, the resultant gas being compressed prior to being exported. The resultant liquids are metered before being routed to medium pressure (MP) and low pressure (LP) Separators in which the Skene fluids are commingled with the Buckland wellstream fluids. The stabilised oil produced from the LP Separator is fiscally metered before run-down to storage cells. The flashed gas from the MP and LP separators is compressed and commingled with other Fields' gas before being exported.

The Skene HP separation process is presented schematically in more detail in Fig. 2:

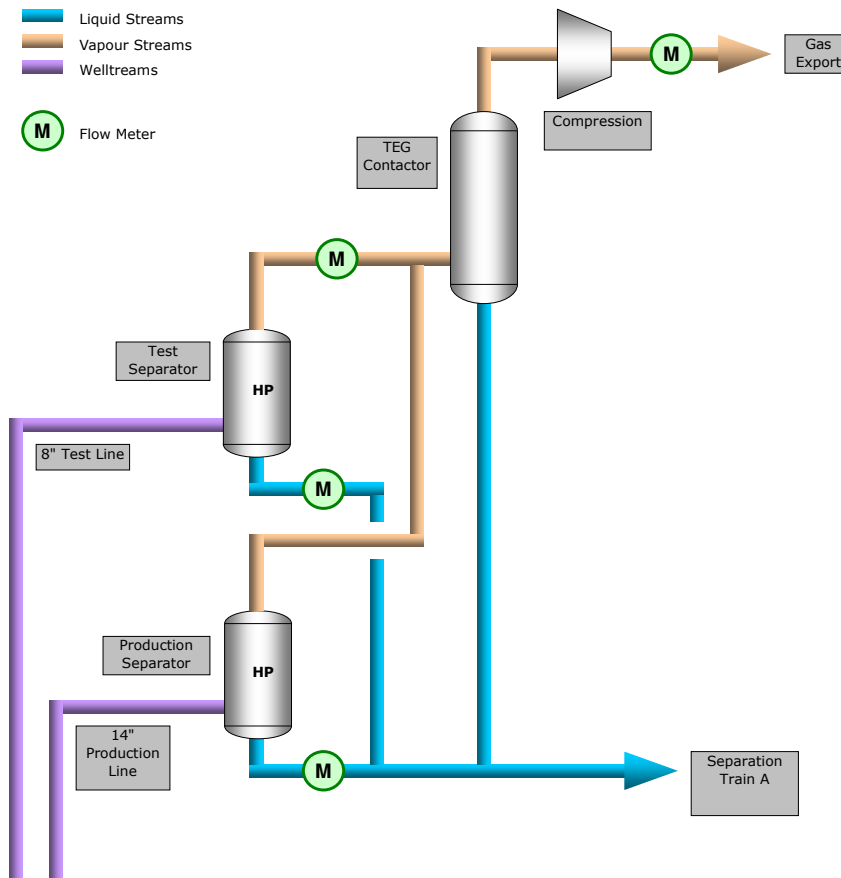


Fig. 2 – Schematic Skene HP Separation and Compression

The Skene wellstream fluids are produced through the two parallel flowlines, referred to as Test and Production, into the Test and Production separators respectively. The Test flow line and associated separator is normally used as a parallel production facility. The vapour from the separators passes through a tri-ethylene glycol (TEG) dehydration contactor, is compressed, and then exported into the Scottish Area Gas Evacuation (SAGE) Pipeline.

The liquids discharged from the Skene Test and Production Separators are volumetrically metered and the water in oil content continuously measured. This liquid stream is routed into Separation Train A where it is commingled with Buckland fluids.

Any liquids condensed in the vapour entering the TEG contactor is commingled with the Skene Separator Oil and also routed into Separation Train A.

The Train A and B Separation Trains are presented schematically in more detail in Fig. 3:

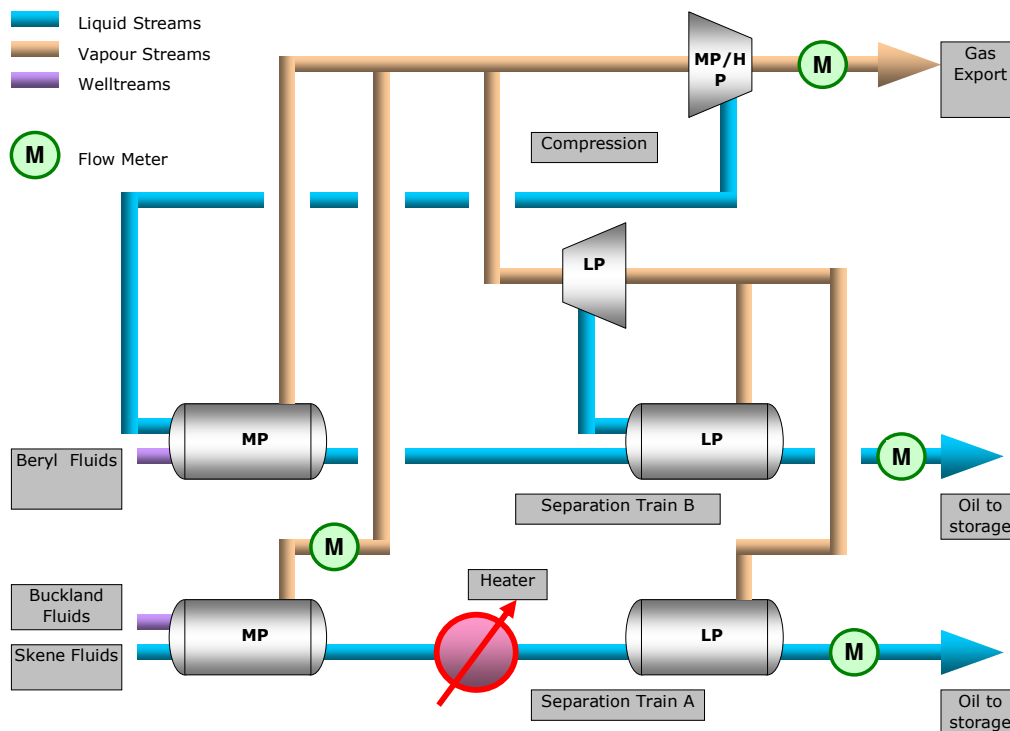


Fig. 3 – Schematic Trains A and B Separation and Compression

In Train A, Buckland and Skene fluids are introduced into the MP Separator where the primary separation takes place. The liquid from the MP Separator is routed to the LP Separator (via a heater) where the separated liquid is stabilised and any remaining vapour flashed off. The liquid is fiscally metered prior to being discharged into the storage cells.

The vapour from the LP and MP Separators is routed into a compression train where it is commingled with gas from further Beryl Fields flashed from the Train B Separators. Any liquid recycles from the compression train are routed back into Train B only.

The gas produced from the compression facilities is consumed as fuel gas, injection gas or exported into the SAGE Pipeline. This compressed gas stream is a commingled mixture of the various Beryl Fields, Buckland Field and Skene Field flash gas.

For process reasons the Buckland wells could not be routed to the platform Test Separator (not shown) and well testing by-difference was used to estimate the individual Buckland well production. Well testing by-difference involves measuring the oil and gas flows with all wells producing then shutting in the well to be tested. The drop in flow rate reflects the flow contribution from that well. The total production from the Buckland Field could be estimated by summing the flow for each well estimated from the by-difference well tests. Well testing by-difference was conducted using the oil meter from the LP Separator and gas meter on the MP Separator in Train A.

### **3 DEVELOPMENT OF THE UNCERTAINTY BASED ALLOCATION SYSTEM**

#### **3.1 Historical**

Prior to the end of 2001 Train A was used to process Buckland fluids exclusively. Therefore, there was no requirement to sub-allocate the oil production from Train A with any other fields. With the introduction of partially stabilised Skene fluids into Train A, allocation between the Skene and Buckland Fields became an important aspect of the allocation system.

Historically the Beryl allocation system (which included sub allocation between Fields in Train B) was based on well tests. For various reasons this approach could not be extended to accommodate the Skene well production and meters were installed to measure the liquid production from the Skene HP Separators.

After allowance for shrinkage, the Skene Field oil production from Train A could be relatively accurately estimated directly from the HP Separator liquid meters.

Allowing for processing effects the Buckland Field oil production could be estimated from the aggregate of the by-difference wells tests. However, this estimate of Field production was considerably less accurate than the Skene Field's for the following reasons:

- The by-difference well tests were intermittent and well decline had therefore to be estimated for the periods between the tests
- Well production on a day was also dependent on well uptime which was calculated based on choke opening and closing and would also therefore be affected by ramp up of flow on opening
- The accuracy of by-difference wells tests is poorer than direct well tests since the uncertainty in two measurements contributes to the overall uncertainty of the by-difference test
- The introduction of the Skene fluids into Train A diminished the accuracy of the by-difference test. This is because, though the relative uncertainty in the two flow measurements remains unchanged, the additional flow renders the absolute uncertainty in the two measurements greater and this larger absolute uncertainty therefore increases the uncertainty in the by-difference calculated flow.

Because of the significant difference between the accuracy of the two fields' estimated oil production – termed field potentials – a variety of allocation options was considered and these are described in the next section.

#### **3.2 Development of Uncertainty Based Allocation Approach**

Two commonly used methods were apparent for allocating the Train A oil between the Skene and Buckland Fields:

- the first was to allocate in proportion to calculated potentials (proportional allocation)
- the second was to allocate Skene its potential and Buckland the difference between the metered Train A Oil product and the Skene potential (by-difference allocation)

The problem with the first, proportional method is that the relatively inaccurate Buckland potential reduces the accuracy of the allocation of Skene Train A Oil especially at lower Skene flows. The second, by-difference method, whilst taking advantage of the more accurate Skene potential, can give significant errors in the Train A Oil allocated to Buckland at low

Buckland flow rates. The proportional allocation method does not take advantage of the more accurate Skene potential, whilst the second, by-difference method, discards a piece of information - the Buckland potential.

A third method (UBA) was therefore developed, that takes advantage of the more accurate Skene potential but also incorporates the Buckland potential and still accurately allocates Buckland at relatively low rates. The method is based on recognised mathematical techniques used in data reconciliation [1] and the resultant allocation equations are straightforward.

The three methods are discussed in Sections 3.3 and 3.4.

### 3.3 Oil Allocation Equations

#### Proportional Method

Allocated quantities employing the proportional method are given by:

$$A_S = V_M * \left( \frac{P_S}{P_S + P_B} \right) \quad (1)$$

and,

$$A_B = V_M * \left( \frac{P_B}{P_S + P_B} \right) \quad (2)$$

#### By-difference Method

Allocated quantities employing the by-difference method are given by:

$$A_S = P_S \quad (3)$$

for Skene and by-difference,

$$A_B = V_M - P_S \quad (4)$$

for Buckland.

#### Uncertainty Based Allocation Method

Allocated quantities employing the UBA method are given by:

$$A_S = P_S + \beta(V_M - P_B - P_S) \quad (5)$$

$$A_B = P_B + (1 - \beta)(V_M - P_B - P_S) \quad (6)$$

$$\beta = \frac{e_S^2}{e_S^2 + e_B^2} \quad (7)$$

These equations have been derived using recognised mathematical techniques. The resultant equations are identical to those presented in the API RP 85 [3] if the uncertainty in the oil product meter is ignored.

The derivation of the method presented in this paper was developed in 2000, independently from any methods subsequently published in API RP 85. In fact the method developed for the Beryl allocation system was termed the "Reconciliation" method during the development phase (and is referred to as such in the allocation agreement) as it was based on data

reconciliation techniques. It only subsequently became apparent that this was essentially the same equation as that more commonly termed Uncertainty Based Allocation and the latter term has therefore been used throughout this paper.

The derivation used in this paper, to arrive at the above equations, is different to that developed for the API. It is interesting to note that essentially the same equation should be developed, using different mathematical approaches, by two independent parties based on an emerging need of the industry. A full derivation of these equations is presented in Section 7.

Inspection of the UBA equations (5 to 7) reveals that, if the uncertainty associated with the Skene potential is very much less than that associated with the Buckland potential (i.e.  $e_s \ll e_B$ ),  $\beta$  tends to zero and the UBA equations reduce to the by-difference equations.

### 3.4 Theoretical Analysis of Allocation Results

#### Simplified Examples

For the case of perfect data, i.e. the potentials summing exactly to the metered oil, all three methods allocate identically, in fact they allocate each field its potential.

For the normal case when potentials do not sum to the metered rate, the impact of uncertainties in the Skene and Buckland potentials can best be demonstrated using simple numerical examples.

In the following examples it has been assumed that the uncertainty of the Skene potential is  $\pm 1\%$  and that of the Buckland potential is  $\pm 5\%$ . (These are representative figures for illustrative purposes but are not the actual values used for the allocation system (see Section 3.5)). This reflects the greater accuracy associated with the Skene potential compared with that of the Buckland potential.

Case 1 illustrates the situation when both fields are producing at similar rates; the allocation results from the three methods are presented in Table 1:

**Table 1 – Comparison Case 1: Similar Potentials**

|                         | Skene | Buckland | Total | Metered |
|-------------------------|-------|----------|-------|---------|
| Metered                 |       |          |       | 100.0   |
| Potential               | 50.5  | 52.5     | 103.0 |         |
| Allocated Proportional  | 49.0  | 51.0     |       |         |
| Allocated By-Difference | 50.5  | 49.5     |       |         |
| Allocated UBA           | 50.4  | 49.6     |       |         |

In this case both potentials are over-predicting the Train A Oil rate. The proportional method 'shares out' the error in the potentials and Skene is allocated a figure that is over 3% below its potential, which is in excess of the uncertainty associated with its potential. The by-difference and UBA methods allocate similar quantities.

Case 2, presented in Table 2, illustrates the situation when Buckland is the dominant flow:

**Table 2 – Comparison Case 2: High Buckland, Low Skene Potentials**

|                         | Skene | Buckland | Total | Metered |
|-------------------------|-------|----------|-------|---------|
| Metered                 |       |          |       | 100.0   |
| Potential               | 5.0   | 100.0    | 105.0 |         |
| Allocated Proportional  | 4.8   | 95.2     |       |         |
| Allocated By-Difference | 5.0   | 95.0     |       |         |
| Allocated UBA           | 5.0   | 95.0     |       |         |

In this case there appears to be about a 5% over-prediction associated with the Buckland potential. The Proportional method causes Skene to be allocated 5% less than its potential but both the by-difference and UBA methods allocate Skene its potential and Buckland receives the difference. An under-prediction associated with Buckland potential would have over-allocated Skene by a similar amount.

Cases 1 and 2 demonstrate the problem associated with the proportional method in that the accuracy of the Skene allocated quantities does not appear to be in accordance with the accuracy associated with its potential.

Case 3, presented in table 3, illustrates the situation when Skene is the dominant flow:

**Table 3 – Comparison Case 3: Low Buckland, High Skene Potentials**

|                         | Skene | Buckland | Total | Metered |
|-------------------------|-------|----------|-------|---------|
| Metered                 |       |          |       | 100.0   |
| Potential               | 96.0  | 5.0      | 101.0 |         |
| Allocated Proportional  | 95.0  | 5.0      |       |         |
| Allocated By-Difference | 96.0  | 4.0      |       |         |
| Allocated UBA           | 95.1  | 4.9      |       |         |

The by-difference method causes Buckland to be allocated 20% less than its potential, even though its potential is known to 5% accuracy. The proportional and UBA methods allocate both fields to within 1% of their potentials. At even higher Skene flows the by-difference method can allocate Buckland negative Train A Oil.

Summarising the three cases:

- for the allocated Skene Train A Oil, the accuracy of the UBA method is as good as the by-difference method and considerably better than the proportional method.
- for the allocated Buckland Train A Oil, the accuracy of the UBA method is comparable with the proportional method and considerably better than the by-difference method especially at low Buckland flows.

### Uncertainty Analysis

To confirm these findings more rigorously, analytical methods [2] have been used to determine the uncertainty associated with the allocation results produced by the three methods over the full range of Skene to Buckland flow ratios.

The following has been assumed in the analysis:

- the Skene Train A Oil potential is known to within +/- 1% being based primarily on a metered figure
- the Buckland Train A Oil potential is known to within +/- 5% being based primarily on well test data
- the metered Train A Oil is known to within +/- 1%.

The uncertainties in the allocated Skene Train A Oil are presented in Fig 4:

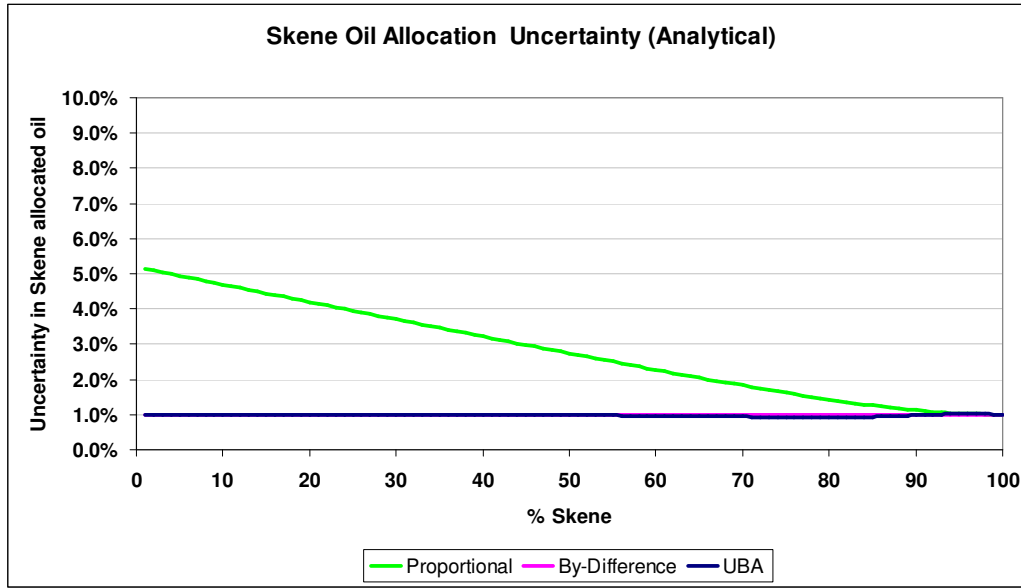


Fig. 4 – Skene Train A Oil Allocation Uncertainty

The horizontal axis represents the percentage of the total Train A Oil flow that Skene comprises. The uncertainty of the Skene allocated Train A Oil is +/- 1% for the by-difference and UBA methods over the whole range of Skene flow. (The loci of the by-difference and UBA lines lie virtually on top of one another). The uncertainty under the proportional allocation scheme increases as the Skene flow reduces because the uncertainty associated with the Buckland potential starts to dominate.

The analogous uncertainties in the allocated Buckland Train A Oil are presented in Fig 5:



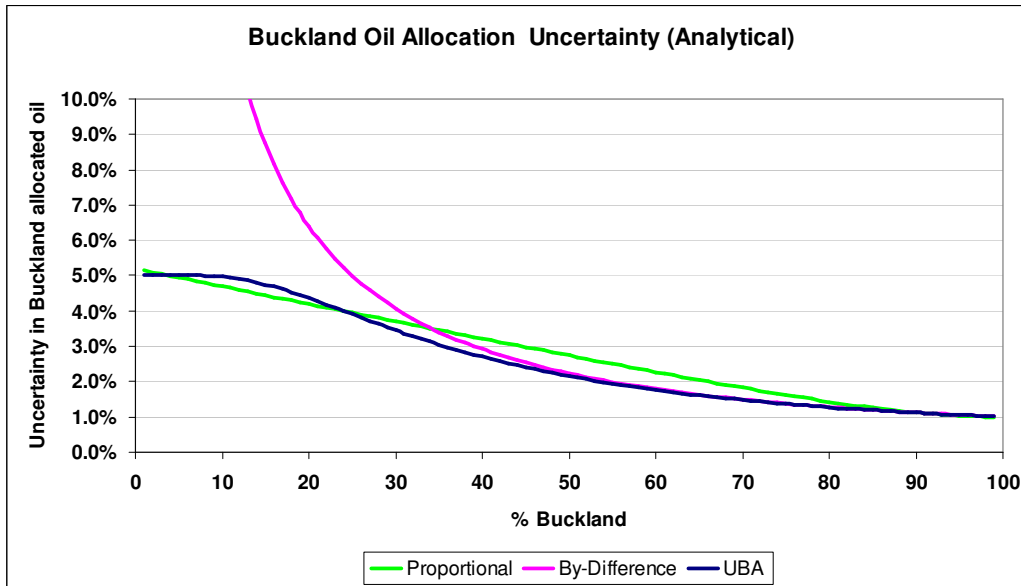


Fig. 5 – Buckland Train A Oil Allocation Uncertainty

In this chart the horizontal axis represents the percentage of the total Train A Oil flow that Buckland comprises. The uncertainty of the Buckland allocated Train A Oil is similar for the proportional and UBA methods over the whole range of Buckland flow. At low flows the by-difference method produces large uncertainties in the Buckland allocated quantity because it is calculated as the small difference between two large numbers.

The above analyses demonstrate that the UBA method appears to be the most accurate of the three methods.

### 3.5 Calculation of Field Potential Uncertainties

#### Buckland

The Buckland field uncertainty was calculated based on 12 months of historical allocation data prior to the start up of Skene. The Buckland combined well potential was compared against the metered oil production and the difference used to estimate the uncertainty in the potential. This calculation accounted for the contribution of the Train A Oil meter uncertainty to the observed difference.

The calculated uncertainty in the Buckland Field potential was 5.8% of flow.

#### Skene

The Skene field uncertainty is made up of a number of sources: flow meters, oil in water meter and shrinkage. The flow and oil in water meter uncertainties were obtained from quoted manufacturers' data. The process shrinkage uncertainty was calculated by combining the individual hydrocarbon component shrinkage uncertainties. It might be anticipated that the shrinkage could be a source of considerable uncertainty but in this instance it was mitigated by the fact that the metered fluid was partially stabilised.

The calculated uncertainty in the Skene Field potential was 0.7% of flow.

#### 4 ALLOCATION RESULTS

##### 4.1 Field Potentials and Allocated Oil

The allocation results for Skene and Buckland are presented in Figs 6 and 7 respectively. The data has been smoothed for reasons of clarity.

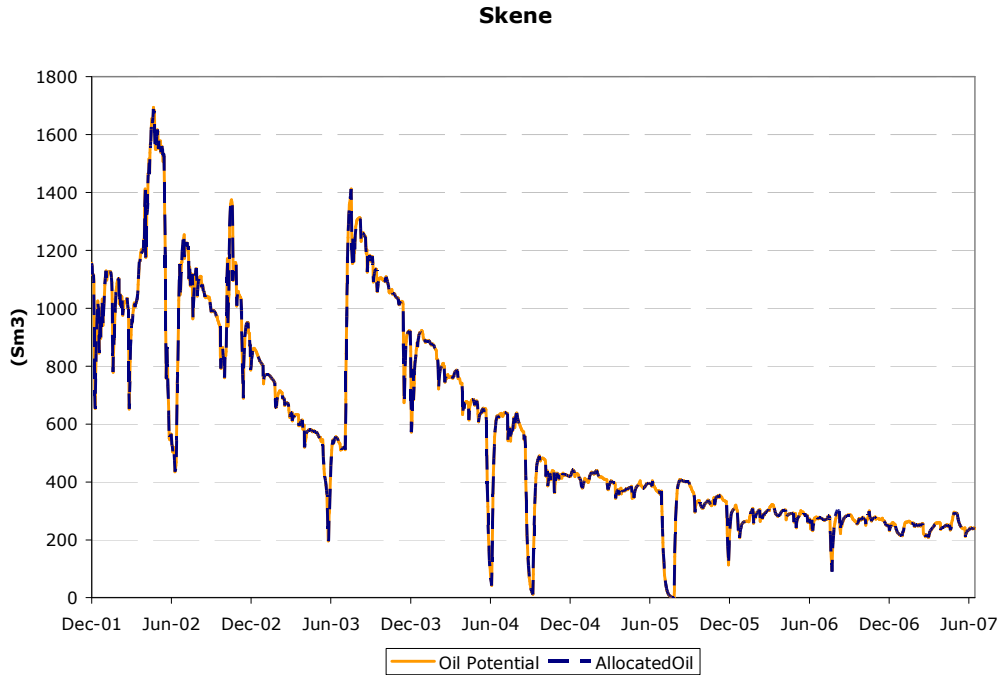


Fig. 6 – Skene Actual Allocated Oil and Potential

The loci of the oil potential and allocated oil lines lie virtually on top of one another.

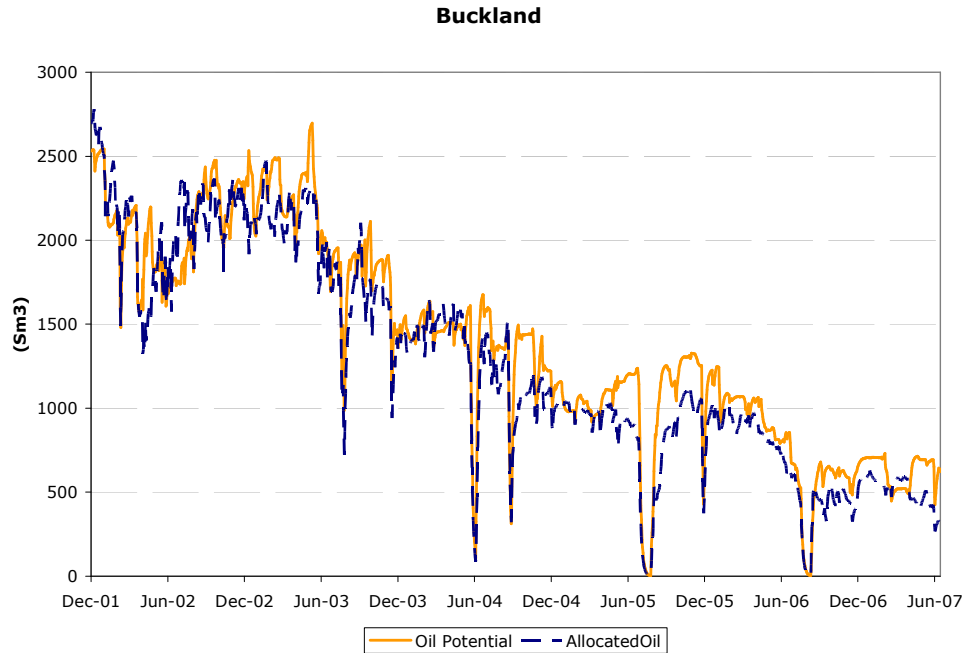


Fig. 7 – Buckland Actual Allocated Oil and Potential

The data presented spans a period of over five and half years (> 2,000 days). During this period the UBA has performed robustly and the allocated quantities have been deemed consistent and accurate.

Because the absolute uncertainty in the Skene Field potential is significantly less than the Buckland Field's, Skene is allocated a quantity very close to its potential and effectively Buckland is allocated the remainder. This is much as expected as the absolute uncertainty in the Skene potential is much less than the Buckland value (i.e.  $e_s \ll e_B$ ), caused by the fact that the relative uncertainty of the Buckland potential is about seven times greater than the Skene value and Buckland represents approximately two thirds of the oil production.

The absolute uncertainty in the Buckland potential would be expected to reduce when the Buckland production is low. This only tends to occur on odd individual days. Fig 8 shows the allocation results for such a day:

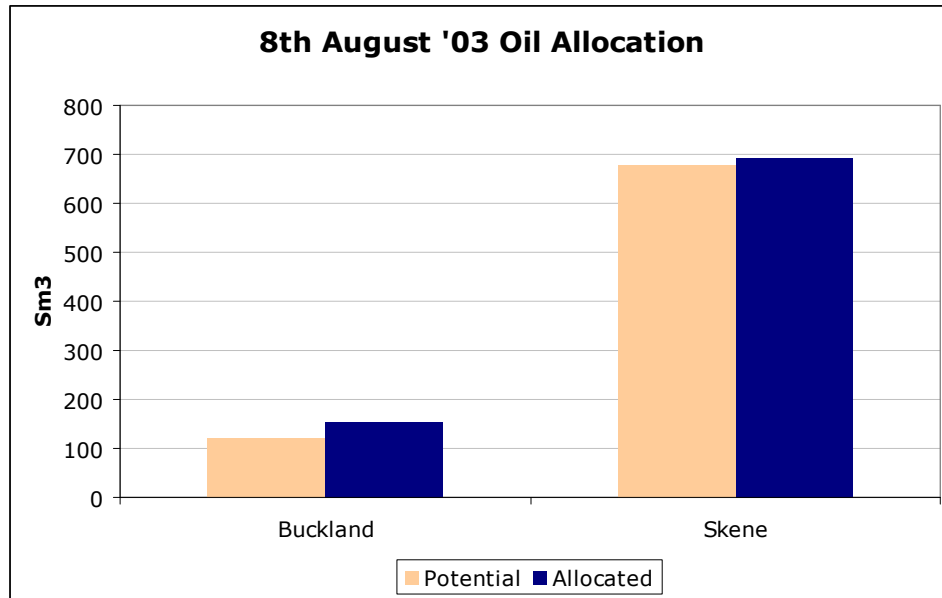


Fig. 8 – Oil Potential and Allocation at Low Buckland Production

The results show that at low flow rates Buckland is allocated more closely to its potential as expected.

#### 4.2 Comparison with Alternative Allocation Results

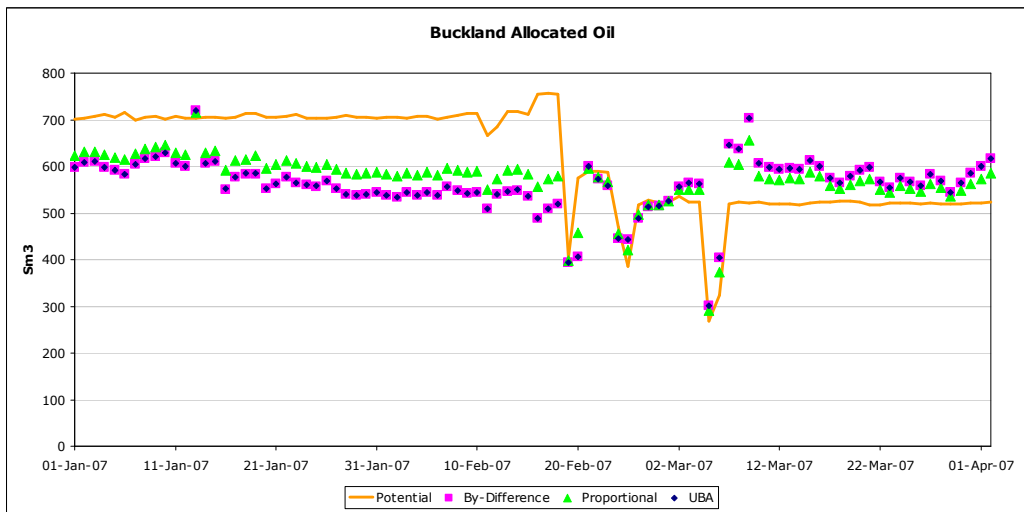


Fig. 9 – Comparison of Allocation Methods - Buckland

This chart shows the Buckland estimated oil potential (solid line) and oil which would be allocated by each of the three allocation methods (indicated by markers) for a three month period in the data set. The by-difference and UBA markers lie virtually on top of one another. The UBA results are as per the actual allocation.

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12-14 February 2008

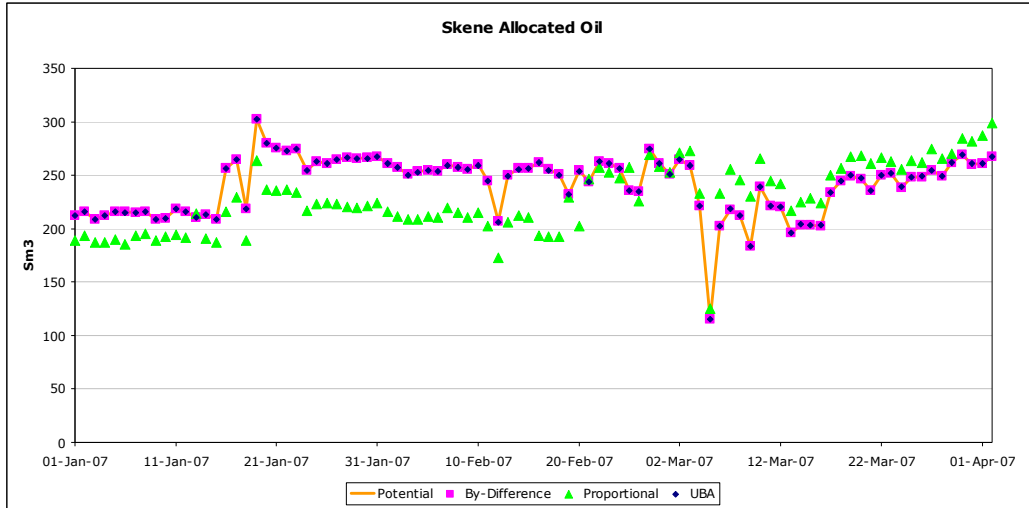


Fig. 10 – Comparison of Allocation Methods - Skene

The data for Skene shows the by-difference and UBA allocated quantities are all virtually the same as the potential.

However, proportional allocation would have allocated Skene significantly below its potential (up to 26% below) in the period prior to mid February '07 and then above its potential from March onwards. This change coincides with a significant decrease in the Buckland potential when the wells were tested. It appears that the decline since the previous Buckland well tests was being under-estimated and hence the Buckland potential over-estimated biasing the proportional allocation in Buckland's favour. The by-difference and UBA methods allocated a quantity of oil to Buckland in February comparable with the Buckland re-tested potential.

Figs 11 and 12 below illustrate how the three different methods of allocation behave when Buckland drops to zero production.

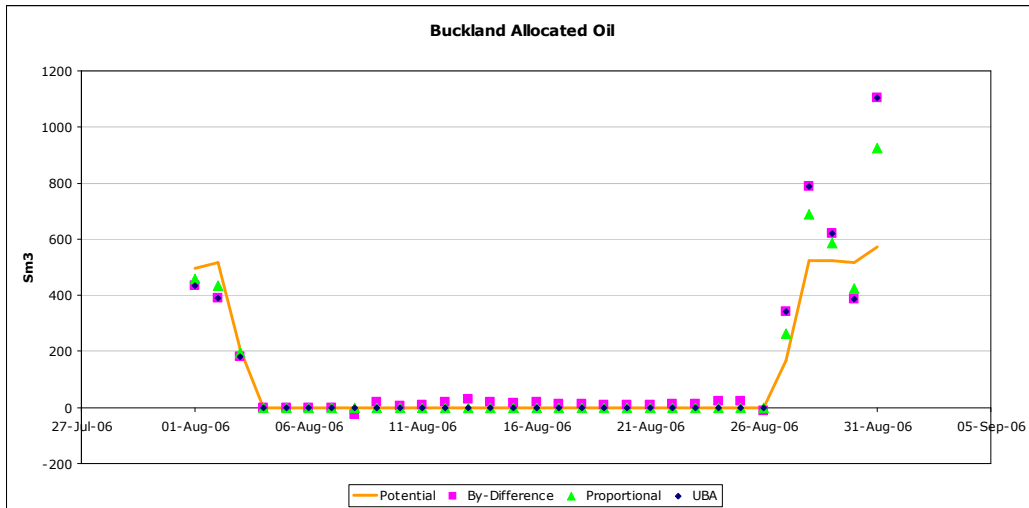


Fig. 11 – Comparison of Allocation Methods During Periods of Shut In – Buckland

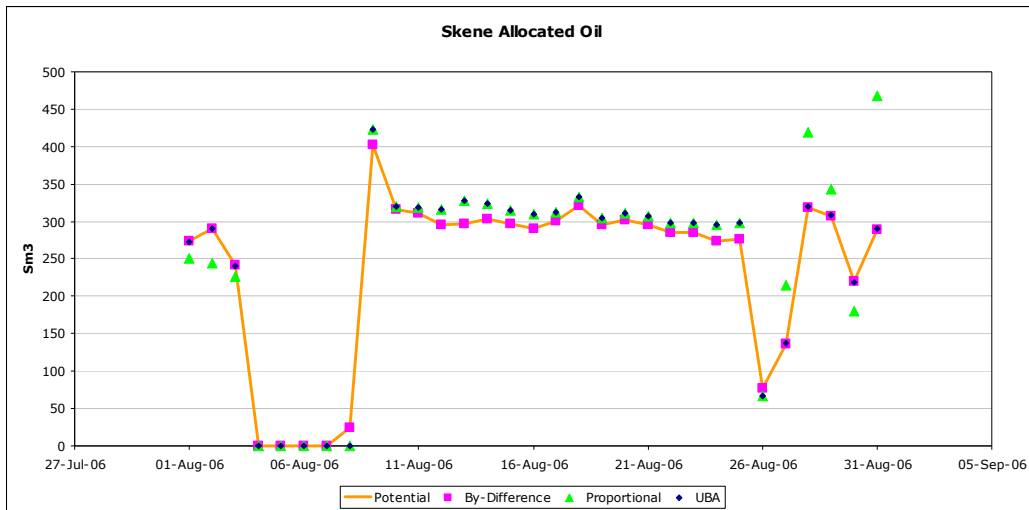


Fig. 12 – Comparison of Allocation Methods During Periods of Shut In - Skene

The by-difference method allocates small quantities of material to Buckland for the majority of the days it is shut in and on one day allocates it a negative quantity. The UBA method correctly allocates Buckland zero oil during the entire shut-in period.

## 5 CONCLUSIONS

A UBA allocation methodology has been developed and successfully implemented in a North Sea offshore allocation system.

The UBA has performed robustly and the allocated quantities have been deemed consistent and accurate over a five and half year period.

The UBA accounts for the uncertainty in the estimated production from the two fields, which are estimated using different methods (meter based versus well test based). During periods of significant Buckland flows, the UBA ensures the relatively large uncertainty in Buckland's potential does not have a deleterious effect on the Skene allocation and ensures Skene is allocated quantities consistent with its more accurately known potential.

The UBA correctly allocates Buckland zero production in the periods when it is shut in, in contrast to the by-difference approach which either allocates it small positive or negative quantities.

## 6 ACKNOWLEDGEMENTS

The authors would like to thank the ExxonMobil North Sea Hydrocarbon Allocation Group for allowing the use of the allocation data.

## 7 MATHEMATICAL DERIVATION OF UBA EQUATION

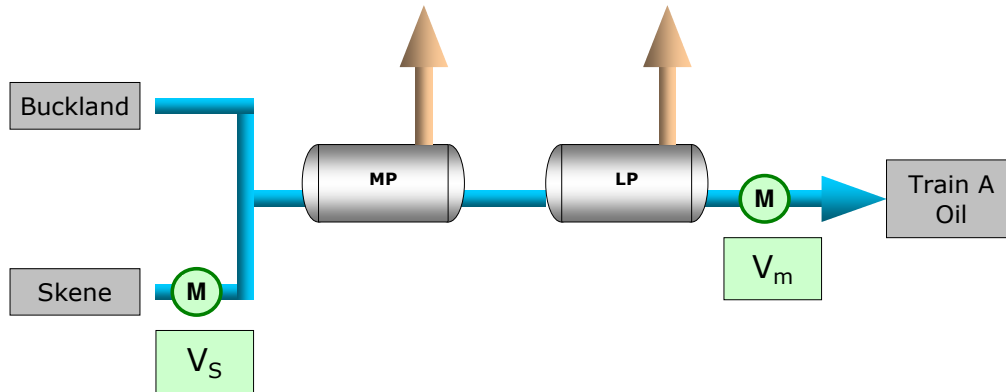


Fig. 13 – Simplified Schematic of Commingled Skene and Buckland Production

The Skene Oil potential ( $P_S$ ) is calculated based on  $V_S$  but compensating for the flashing off of the vapour in the MP and LP separators.

The Buckland Oil potential ( $P_B$ ) is estimated based on well tests (by-difference) and compensating for differences in separator operating temperatures between the well test day and the allocation day.

It is a requirement that the allocated volumes must sum to the metered Train A Oil volume:

$$A_S + A_B = V_M \quad (8)$$

but in general the potentials will not sum to the metered Train A Oil volume:

$$P_S + P_B \neq V_M \quad (9)$$

The UBA method adjusts the potentials, to meet the constraint that the sum of the allocated volumes equals the metered volume, in such a fashion that the sum of the squares of the differences between potential and allocated numbers is minimised. In fact these differences are weighted, based on the uncertainties associated with the potentials.

The sum of the weighted squares of these differences is made equal to a variable E:

$$E = \left( \frac{A_S - P_S}{e_S} \right)^2 + \left( \frac{A_B - P_B}{e_B} \right)^2 \quad (10)$$

Hence, the UBA method can be expressed as minimising the value of E whilst complying with the constraint that the allocated Train A Oil volumes equal the metered value ( $V_M$ ).

This constrained minimisation problem can be solved using the method of Lagrangian multipliers. Re-expressing the constraint equation as a function of  $\Phi$ :

$$\Phi = A_S + A_B - V_M \quad (11)$$

The method of Lagrangian multipliers employs the following equations to solve for  $A_S$  and  $A_B$ :

$$\left( \frac{\partial E}{\partial A_S} \right) + \lambda * \left( \frac{\partial \Phi}{\partial A_S} \right) = 0 \quad (12)$$

$$\left( \frac{\partial E}{\partial A_B} \right) + \lambda * \left( \frac{\partial \Phi}{\partial A_B} \right) = 0 \quad (13)$$

$$A_S + A_B - V_M = 0 \quad (14)$$

Differentiating:

$$\left( \frac{\partial E}{\partial A_S} \right) = 2 * \frac{(A_S - P_S)}{e_S^2} \quad (15)$$

$$\left( \frac{\partial E}{\partial A_B} \right) = 2 * \frac{(A_B - P_B)}{e_B^2} \quad (16)$$

$$\left( \frac{\partial \Phi}{\partial A_S} \right) = 1 \quad (17)$$

$$\left( \frac{\partial \Phi}{\partial A_B} \right) = 1 \quad (18)$$

Substituting from Equations 15 to 18 in Equations 11 and 12 to give:

$$2 * \frac{(A_S - P_S)}{e_S^2} + \lambda = 0 \quad (19)$$

$$2 * \frac{(A_B - P_B)}{e_B^2} + \lambda = 0 \quad (20)$$

Subtracting Equation 19 from 20 to eliminate  $\lambda$  and obtaining  $A_B$  in terms of  $A_S$  from Equation 14 gives the following in  $A_S$ :

$$\frac{(V_M - A_S - P_B)}{e_B^2} - \frac{(A_S - P_S)}{e_S^2} = 0 \quad (21)$$

Which can be rearranged to give the following in terms of  $A_S$ :

$$A_S = P_S + \left( \frac{e_S^2}{e_S^2 + e_B^2} \right) (V_M - P_B - P_S) \quad (22)$$



Similarly for  $A_B$ :

$$A_B = P_B + \left( \frac{e_B^2}{e_S^2 + e_B^2} \right) (V_M - P_B - P_S) \quad (23)$$

Letting,

$$\beta = \frac{e_S^2}{e_S^2 + e_B^2} \quad (24)$$

Allows 22 and 23 to be expressed:

$$A_S = P_S + \beta(V_M - P_B - P_S) \quad (25)$$

$$A_B = P_B + (1 - \beta)(V_M - P_B - P_S) \quad (26)$$

## 8 NOTATION

|       |  |           |                                |
|-------|--|-----------|--------------------------------|
| $A_B$ | allocated Buckland Train A Oil             | $P_B$     | Buckland Train A Oil potential |
| $A_S$ | allocated Skene Train A Oil                | $P_S$     | Skene Train A Oil potential    |
| $E$   | optimised variable                         | $V_M$     | metered Train A Oil            |
| $e_S$ | absolute uncertainty in Skene potential    | $V_S$     | metered Skene Liquids          |
| $e_B$ | absolute uncertainty in Buckland potential | $\beta$   | uncertainty ratio variable     |
|       |  | $\lambda$ | Lagrangian multiplier          |
|       |  | $\Phi$    | constraint variable.           |

## 9 REFERENCES

- [1] Data Processing and Reconciliation for Chemical Process Operations, J. Romagnoli and M Sanchez, Published by Academic Press (2000), ISBN 0-12-594460-8.
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