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Technical Paper

Measuring Flow Without A Meter

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1 INTRODUCTION

1.1 Overview

This paper concerns maximising the use of available data to reduce measurement costs.

Consider the following processing scenario depicted in the schematic, Figure 1:

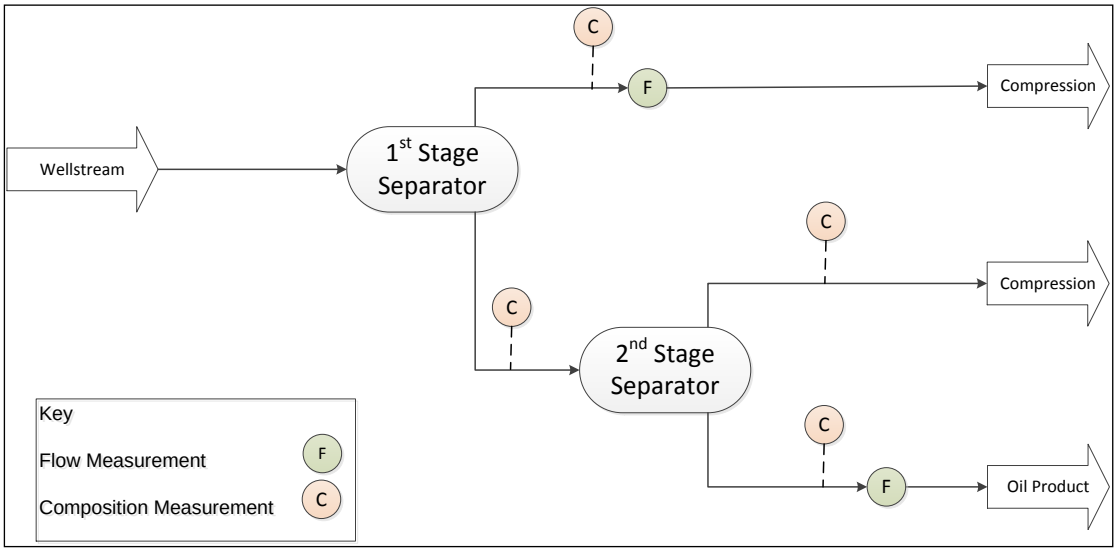

Figure 1 – System Configuration

A wellstream is being produced through two stages of separation. A knowledge of the flow and composition of the wellstream is required for allocation purposes.

The 1st stage gas and 2nd stage oil mass flow rates are metered but the flow of gas from the 2nd stage is not. Hence, at first sight there appears to be insufficient information to calculate the wellstream mass flow and composition, since only two out of the three stream discharge streams are known.

However, sample points are available on both separators' gas and oil discharge streams and hence their compositions can be determined. With a knowledge of these compositions and only two out of three flow measurements, is it possible to determine the wellstream flow and composition and consequently save the cost of installing a meter on the 2nd stage gas stream?

This paper illustrates that there is in fact sufficient data to infer both the flow and composition of the wellstream. In fact, the calculation approach is straightforward in that it simply uses a knowledge of the process topology and mass/molar balances across the process units.

The paper goes on to calculate typical uncertainties in the calculated wellstream figures. This allows a cost benefit analysis to be performed on the installation of a meter in the 2nd stage gas stream.

The above scenario is based on a real process and the calculations were developed in conjunction with Apache North Sea for the inference of the flow and composition of a new tie back, the Callater Field, to their Beryl Alpha platform. The data presented in the paper has been adjusted and anonymised for commercial confidentiality reasons, but it is still realistic and representative of the actual production.

The approach developed in conjunction with Apache was termed the "Inferential Method" and is used throughout the paper to refer to the method of calculation developed to infer the flow and composition of the well stream without the 2nd Stage Gas Meter. In contrast, the term "Direct Measurement" is used to refer to the case in which the wellstream is calculated assuming the 2nd Stage Gas Meter is installed.

1.2 Structure of the Paper

Section 2 describes the process configuration and the data available. It also explains why the installation of a new 2nd Stage Gas Meter is problematic and therefore undesirable.

Section 3 provides an overview of the inferential calculation methodology (though a more complete mathematical derivation is presented in Section 8) and illustrates how it works using a simplified example. The text goes on to extend the method to incorporate a simple flash calculation which eliminates the requirement for one of the stream compositions to be physically sampled and hence reduce costs further.

A calculation of the uncertainty associated with the Inferential Method compared with that of the Direct Method is presented in Section 4. This then feeds into a cost benefit analysis for the installation of the 2nd Stage Gas Meter discussed in Section 5.

Section 6 presents an alternative, more generalised approach to the Inferential Method which more readily determines its applicability to other potentially more complex systems.

Section 7 provides conclusions.

2 DESCRIPTION OF THE SYSTEM AND MEASUREMENT ISSUE

The Callater wellstream was to be produced through two stages of separation on Beryl Alpha. The process is represented in the schematic, Figure 2:

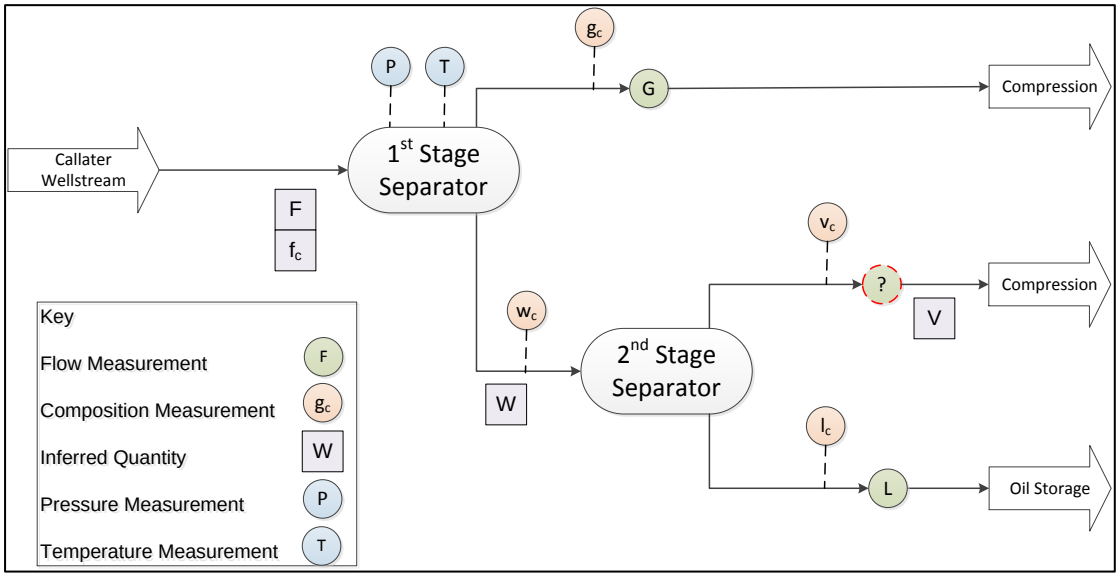

Figure 2 – Beryl Alpha Separator Train Configuration

The separators and measurement points were all existing prior to the tie-back of Callater.

The existing measured flows are indicated by green circles, measured compositions by orange circles and all unmeasured quantities (to be inferred) by purple squares. The potential future 2nd Stage Gas Meter is indicated by the red, dashed bordered green circle containing a question mark.

The flow (F) and composition (f_c) of the wellstream are required for allocation purposes. The 1st stage gas (G) and 2nd stage oil (L) mass flow rates are measured as are the 1st stage gas (g_c) and oil (w_c) and 2nd stage gas (v_c) and oil (l_c) compositions. Ideally, the wellstream flow is obtained directly from:

$$F = G + V + L \tag{1}$$

And the composition from:

$$f_c = \frac{Gg_c + Vv_c + Ll_c}{G + V + L} \tag{2}$$

These are the equations that would be used in the Direct Method if the 2nd Stage Gas Meter was installed.

However, the flow of gas (V) from the 2nd stage is not metered. Hence, there appears to be insufficient information to calculate the Callater wellstream mass flow and composition.

Problematically, there were significant technical and associated cost issues with the retro-fitting of a new flow meter in the gas discharge line from the 2nd Stage Separator. These issues included:

- a low differential pressure across the meter would mean that selection of a suitable meter would be difficult;

- the uncertainty of the measurements from such a meter would consequently be increased;

- there would be significant piping and module modifications required to situate the meter in a suitable location, resulting in significant installation costs.

The above factors stimulated the investigation into possible alternative methods of determining the wellstream composition and flow.

As already mentioned, sample points are available on both separators' gas and oil discharge streams and hence their compositions could be determined. The question was posed that: "with knowledge of these compositions and only two out three flow measurements, is it possible to determine the wellstream flow and composition and consequently save the cost of installing a meter on the 2nd stage gas stream?"

The subsequent analysis revealed that it was possible to determine the Callater wellstream using only the existing flow and sample data already available, this was termed the Inferential Method. The calculations used in the Inferential Method are described in the next section.

3 INFERENCE OF THE WELLSTREAM FLOW AND COMPOSITION

3.1 Equations

The derivation of the equations used to infer the wellstream flow and composition are presented in Section 8. The resultant final equations are reproduced below as a series of calculation steps using the available measurement and flow data as starting points.

First a factor (α) is calculated from the available compositions:

$$\alpha = \frac{\sum_c \frac{g_c l_c}{w_c} - 1}{\sum_c \frac{g_c l_c}{w_c} - \sum_c \frac{g_c v_c}{w_c}} \tag{3}$$

α is in fact equal to the ratio of the 2nd stage gas flow to the 1st stage oil flow (V/W). Once α is determined, incorporation of the measured 1st stage gas (G) and 2nd stage (L) oil flows allows the wellstream composition (f_c) to be determined from:

$$f_c = l_c\left(1+\alpha\left(\frac{v_c}{l_c}-1\right)\right)\left(1+\left(\frac{G(1-\alpha)}{G(1-\alpha)+L}\right)\left(\frac{g_c}{w_c}-1\right)\right) \tag{4}$$

And the 1st stage liquid flow (W) is obtained from:

$$W = \frac{L}{1-\alpha} \tag{5}$$

Finally, the wellstream total feed flow (F) is given simply by:

$$F = G + W \tag{6}$$

Though not explicitly required the 2nd stage gas (V) is similarly obtained from:

$$V = W - L \tag{7}$$

Hence, the desired unknown wellstream quantities have been calculated.

3.2 Simplified Example

To illustrate that the method works numerically, a simplified example is presented below. Only three components are included to improve the clarity of the calculations. The flows and compositions are presented as though in molar terms, though they could equivalently be read as mass units. Though fictitious and simplistic the example proves the method works and the limited amount of numerical data presented allows the reader to readily confirm the calculations independently.

The example assumes the same configuration of separators and measurements as presented in Figure 2. The true flows in each of the streams are presented in Table 1:

Table 1 – True Molar Component Flows

Component	Wellstream	1st Stage Gas	1st Stage Liquid	2nd Stage Gas	2nd Stage Liquid
C_1	69	58	11	10	1
C_2	47	20	27	7	20
C_3	84	2	82	3	79
Total	**200**	**80**	120	20	**100**
Stream ID	F	G	W	V	L

The stream identifiers in the bottom row are in accordance with Figure 2. As can be observed there is a perfect molar balance at a total and component level through the process, hence the use of the term "true" flows with reference to these values.

In practice, we would not have direct access to all these figures and would have to calculate them from the available measurements. In the scenario being considered, the 1st stage gas (G) and 2nd stage liquid (L) total flows are measured and these are highlighted with purple text. One objective of the calculations is to determined F highlighted in red. As can be observed, F is not directly obtainable as it would require either V or W to be measured.

Additional measurements in the form of stream compositions are available and these are the purple figures in Table 2:

Table 2 – True Molar Compositions

Composition ID	f_c	g_c	w_c	v_c	l_c
C_1	34.5%	72.5%	9.2%	50.0%	1.0%
C_2	23.5%	25.0%	22.5%	35.0%	20.0%
C_3	42.0%	2.5%	68.3%	15.0%	79.0%

The molar percentage concentrations are consistent with the component flows in Table 1. The red values indicate the wellstream composition which is the second objective of the calculations. Again, this would be easily calculable from the available compositions, if either V or W were known.

The problem is to utilise the four measured compositions and two measured total flows to calculate the wellstream flow F and composition f_c.

The methodology described in Section 3.1 is applied to the available (purple) data values from Table 1 and Table 2.

First α is calculated according to Equation (3), the terms of which are calculated in Table 3:

Table 3 – Calculation of α

	$g_c * l_c / w_c$	$g_c * v_c / w_c$
C_1	0.0791	3.9545
C_2	0.2222	0.3889
C_3	0.0289	0.0055
Sum	0.3302	4.3489
α	**0.1667**	

The wellstream composition f_c, can be obtained from Equation (4), W from (5) and finally F from (6) as presented in Table 4:

Table 4 – Calculation of Wellstream Flow and Composition

	f_c
C_1	34.5%
C_2	23.5%
C_3	42.0%
W	120
F	**200**

As can be seen the calculated wellstream flow and composition, obtained from the measured data values only, correspond with the true values presented in Table 1 and Table 2.

The method is extensible to any number of components and conversions to a mass basis can be performed using component molecular weights.

3.3 Extension of the Method to Incorporate Process Simulation

It is possible to extend the method to incorporate process simulation calculations. In the real system, this was driven by the lack of availability of the 1st stage liquid composition (w_c) which was found to be difficult to obtain in practice.

As indicated in Figure 2, the temperature (T) and pressure (P) of the 1st Stage Separator are measured. With a knowledge of the feed composition (f_c) and the temperature and pressure, the composition of the liquid and vapour streams can be predicted using an appropriate equation of state (such as Peng Robinson or Soave Redlich Kwong) and a standard flash calculation as described in [1]. In addition, the ratio of the gas (G) to the liquid stream (W) total flow rates are also calculated as part of the flash calculation. Since G is measured, W can be determined using this ratio and hence F determined. This calculation is readily performed using a process simulation package such as HYSYS [2] or CHARM [3].

However, f_c is not known and is in fact one of the objectives of the calculation. A way forward is possible however, employing an iterative procedure.

An initial feed composition f_c is guessed and using the temperature and pressure in the 1st Stage Separator, the composition of the separator liquid (w_c) is calculated using the flash unit operation representing the separator in the process model. α can then be determined and a new feed composition calculated from (4). The process iterates until the change in the feed composition is less than a specified tolerance.

The replacement of a measured composition with a process simulation calculation will increase the uncertainty in the method. The uncertainty in the Inferential Method compared with the Direct Measurement approach has not been addressed thus far and this is the subject of the next section.

4 CALCULATION OF UNCERTAINTY

4.1 How Accurate is the Inferential Method in Practice?

The simple example above illustrates and demonstrates the viability of the method but does not provide any information on the uncertainty in the calculated quantities and how this compares with the uncertainty obtained if a meter was installed in the 2nd stage gas line.

Hence an uncertainty analysis was carried out to compare the Inferential and Direct Methods.

The information presented in the next sections has been anonymised but is based on real data, representative of the Callater production over Beryl Alpha.

4.2 Representative Measurement Data

In advance of Callater actually commencing production, the approach employed was to construct a set of representative 1st and 2nd stage gas and oil flows, and associated compositions using a process simulation. This was performed, based on the available wellstream compositional data from PVT (Pressure, Volume, Temperature) reports, for a number of profile years (see Table 12 in Section 5.2).

Generated compositions for the 1st stage gas and 2nd stage gas and liquid, along with the associated flows in molar, mass and standard volume bases, for Year 1 of the profile, are presented in Table 5:

Table 5 – Measured Data Available – Year 1

Compositions		1st Stage Gas		2nd Stage Gas	2nd Stage Liquid
Stream ID		g_c		v_c	l_c
		mole%		mole%	mole%
N_2		1.25%		0.11%	0.00%
CO_2		5.41%		4.13%	0.06%
C_1		72.86%		19.44%	0.10%
C_2		11.98%		17.10%	0.46%
C_3		5.91%		26.19%	2.35%
iC_4		0.50%		4.29%	0.95%
nC_4		1.36%		14.35%	4.34%
iC_5		0.21%		3.25%	2.42%
nC_5		0.25%		4.46%	4.33%
C_6		0.22%		5.08%	15.37%
C_{7+}		0.06%		1.61%	69.62%
Flows					
Stream ID		G		V	L
kmol/d		31,529		2,324	8,241
te/d		704		105	1,275
Sm3/d		745,491		54,953	1,590

The total 2nd stage oil production of 1,275 te/d is equivalent to 10 mbpd and the 809 te/d of total gas produced is equivalent to 28 mmscfd in accordance with Year 1 of the profile presented in Table 12 (Section 5.2).

The 2nd stage gas flows are highlighted in red because these measurements won't be available for the Inferential Method but will be available for the Direct Measurement case (in which the 2nd Stage Gas Meter is installed).

The separator operating temperatures and pressures are presented in Table 6:

Table 6 – Separator Temperatures and Pressures

Vessel	1st Stage Sep	2nd Stage Sep
Temperature (°C)	14	38
Pressure (bara)	11.4	1.2

The 1st Stage Separator conditions are only relevant for the Inferential Method as they are required for the process simulation to calculate w_c.

4.3 Measurement Uncertainties

The flow meter uncertainties used in the analysis are given in Table 7:

Table 7 – Flow Meter Relative Uncertainties

Stream	1st Stage Gas	2nd Stage Gas	2nd Stage Liquid
Stream ID	G	V	L
Uncertainty (±)	1.0%	2.5%	0.25%

The higher uncertainty associated with the 2nd Stage Gas meter is due to the issues described in Section 2.

The gas and liquid compositional uncertainties were estimated based on the variation observed in the composition of multiple repeated gas and liquid samples of the Callater fluids. These calculated uncertainties therefore incorporate variation due to both the sampling procedure and the compositional analysis and are presented in Table 8 in absolute terms:

Table 8 – Compositional Absolute Uncertainties

Compositions	Gas Sample	Liquid Sample
	Abs. Uncert.	Abs. Uncert.
	±mole%	±mole%
N_2	0.07%	0.0001%
CO_2	0.08%	0.03%
C_1	1.0%	0.1%
C_2	0.2%	0.2%
C_3	0.2%	0.5%
iC_4	0.03%	0.2%
nC_4	0.05%	0.8%
iC_5	0.01%	0.4%
nC_5	0.01%	0.6%
C_6	0.03%	1.7%
C_{7+}	0.01%	0.4%

It should be noted that in fact there are covariance terms associated with the compositional uncertainty due to normalisation (to sum to 100%).

The absolute uncertainties of the separator temperatures and pressures are presented in Table 9:

Table 9 – Separator Temperatures and Pressures

Vessel	1st Stage Sep	2nd Stage Sep
Temperature (±°C)	3	3
Pressure (±bara)	0.5	0.1

The accuracy with which temperatures and pressures in vessels can be measured is better than indicated by the uncertainties quoted above. However, the reported temperature and pressures are averages from some period of operation and the

uncertainty in these values is therefore dominated by the process variability. The above values are based on the authors' experience taking account typical process variability as described in [1].

The Inferential Method also includes a process simulation used to calculate the 1st stage liquid composition. The properties of the components, that form part of the inputs to the equations of state used to calculate the thermodynamic vapour liquid equilibria in vessels, will also have an uncertainty. The values described in the Process Simulation Uncertainties paper from the 2014 North Sea Flow Measurement Workshop [1] have been assumed.

4.4 Product Values

To understand the associated impact on the uncertainty in the dollar value of the Callater hydrocarbons produced from the Beryl Alpha platform, the following product values have been used:

Table 10 – Product Values

Product	Units	Value
Gas	£/therm	0.50
LPG	$/te	231
Naphtha	$/te	402
Gas Oil	$/te	423
Vacuum Gas Oil	$/te	321
Residual Fuel Oil	$/te	140

These are based on representative 2016 prices quoted in Platt's Oilgram (this was when the analysis was conducted) and a typical, nominal UK gas price.

4.5 Calculation of Callater Wellstream Flow and Composition Uncertainties

Based on the typical values of the input parameters and their associated uncertainties presented in Sections 4.2 to 4.3 above, a Monte Carlo simulation was run to calculate the uncertainty in the calculated wellstream flow and composition obtained using both the Inferential and Direct Methods.

Monte Carlo Method (MCM), which is described in a Supplement to the GUM [4] and by Coleman and Steele [5], has been employed. The basic methodology is described in [1].

The calculated uncertainties in the wellstream feed flow rate and its associated oil and gas product dollar value are compared for the two approaches in Table 11:

Table 11 – Wellstream Flow and Value Uncertainty

		Case Data	Inferential Method	Direct Method
Feed Flow	(tonnes/d)	2,080		
Uncertainty (abs)	(tonnes/d)		11.5	8.2
Uncertainty (rel)	(± %)		0.55%	0.40%
Value	($/d)	706,631		
Uncertainty (abs)	($/d)		3,281	2,746
Uncertainty (rel)	(± %)		0.46%	0.39%

The above values are for the Year 1 case in which the feed flow slightly exceeds 2,000 tonne/d. The equivalent dollar value of the components is estimated to be just over 700,000 $/d. The uncertainty in the feed flow is slightly greater for the Inferential Method compared with the 2nd Stage Gas Meter Method though only by ±0.15%. When calculated on a dollar value basis this differential reduces to ±0.07%.

The uncertainties associated with the two methods are relatively close because the calculated feed flow is dominated by, and hence principally derived from, the 1st stage gas and 2nd stage oil measurements which are common to both methods.

These uncertainty calculations allow the associated exposure to loss to be calculated and affords a route to a cost benefit analysis of the 2nd Stage Gas Meter.

5 COST BENEFIT ANALYSIS

5.1 Exposure to Loss

Callater experiences an increase in its wellstream dollar value uncertainty with the Inferential Method when compared with the Direct Measurement case. This increased uncertainty will result in an increased exposure to both gain and loss of hydrocarbons. It should be noted that the probability of a certain level of loss is exactly counterbalanced by an equal probability of an equal gain.

However, it is the exposure to loss alone that is considered in the cost benefit analysis. It is possible to multiply each lost revenue figure by its individual probability of occurrence. These can then be summed, or integrated, to give a total risked exposure to loss (E) which is calculated by the following equation:

$$E = \frac{\$U}{\sqrt{8\pi}} \tag{8}$$

Where, $U is the uncertainty (at the 95% confidence level) in the dollar value of the wellstream hydrocarbons.

The full derivation of this equation is provided in [6]; the approach is also referenced in the OGA Measurement Guidelines [7] and by other workers conducting cost benefit analyses [8], [9], [10] and [11].

Using this approach, a cost benefit analysis can be performed in which the additional CAPEX of the 2nd Stage Gas meter can be compared against the reduction in the integrated exposure to loss experienced with the Direct Measurement compared with the Inferential Method.

5.2 Production Profile

To conduct the cost benefit analysis, the production over the anticipated field life is required. The flows and compositions presented in Section 4 are representative of Year 1 of production. Table 12 provides five years of profile data upon which the cost benefit analysis was based.

Table 12 – Production Profile

	Oil		Gas	
Year	bpd	Sm^3/d	mmscfd	mcm/d
1	10,000	1,590	28	0.80
2	5,000	795	40	1.14
3	2,000	318	27	0.77
4	1,000	159	14	0.38
5	700	111	9	0.27

5.3 Monte Carlo Simulation Results

The increases in integrated exposure to loss, experienced with the Inferential Method compared with the Direct Measurement approach, have been calculated using the daily production rates for each profile year and the results presented in Table 13:

Table 13 – Increase in Exposure to Loss – Inferential versus Direct Measurement

	Increased Exposure to Loss	
Year	$/d	$/y
1	$107	$38,988
2	$228	$83,347
3	$190	$69,281
4	$95	$34,517
5	$66	$24,243
Total		$250,376

The values are expressed in $/day for each year and multiplied by 365 to get the equivalent yearly increased exposure (i.e. assuming 100% uptime). Over the complete profile the increased exposure to loss just exceeds $250,000.

5.4 Cost Benefit Calculation

The increased exposure to loss figure of $250,000 can be compared with the installation cost of the new 2nd Stage Gas Meter. In this case, because of the problems highlighted in Section 2 associated with the meter installation, the CAPEX considerably exceeded this figure.

Based on the cost benefit analysis results, the meter was not installed and the Inferential Method has been successfully employed since the start-up of the Callater field in 2017. The savings to the project ran into multiple $100,000's.

6 GENERALISED APPROACH

6.1 Introduction

In practice, the Inferential Method described in the previous sections was developed by manipulating mass component balance equations utilising the available measured data. Before the method was developed it was not obviously apparent that a solution was feasible.

The question arises whether a more systematic approach is available that would determine whether unmeasured stream flows and compositions could be determined from other measured streams and a series of molar/mass balance constraints? Indeed, this would allow the approach to be extended to more complicated processes in which it would not be practicable to determine which streams can de deduced by experimentation.

This type of problem is encountered in the field of data reconciliation in which "observability" is a defined term; indeed, [12] states:

"A variable is said to be observable if it can be estimated by using the measurements and steady state process constraints".

Other papers concerned with observability for both linear and non-linear constraint problems are described and referenced in [12]. The observability problem addressed in this paper is at first sight a non-linear one, since compositions are multiplied by flow rates in the component balance equations (see Equation (21) in Section 8).

However, the constraint equations can be rearranged such that they are linear with respect to the unknown variables. This linearization renders the observability calculations to be more tractable and more easily solvable using matrices.

Based on the methods described in [12], an alternative approach to the inferential calculations has been developed. Its application to the Callater two separator process discussed in this paper is described in the next section.

6.2 Generalised Formulation of Two Separator Scenario

For each node or sets of nodes, mass/molar balances can be written. Here a node is a separator, i.e. a point where a number of streams enter and leave. For example, one such node is the 1st Stage Separator as depicted in Figure 3:

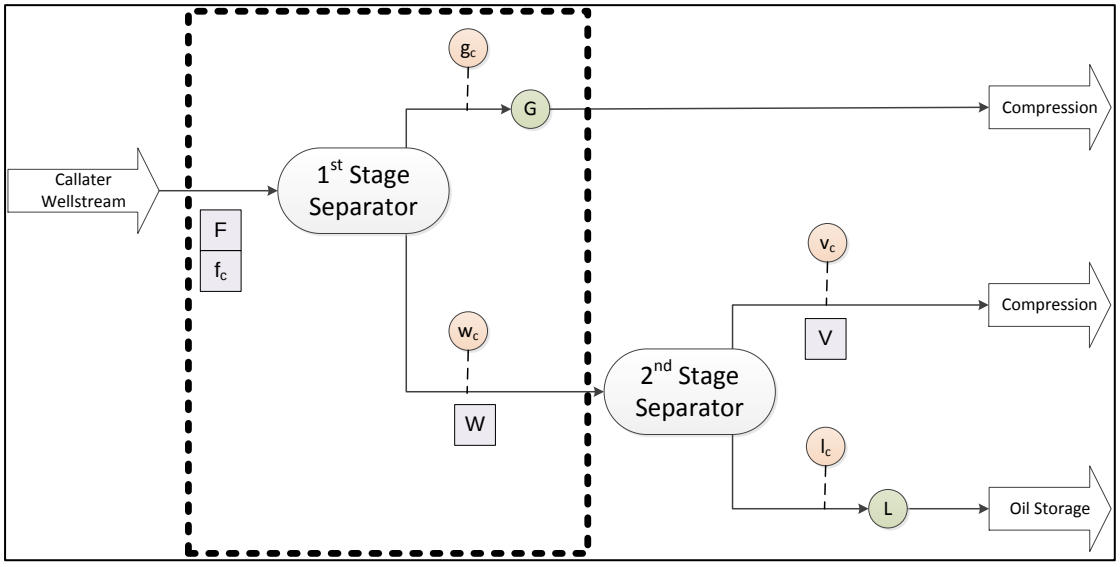

Figure 3 – Process Schematic Process Balance 1

A control surface, indicated by the dashed rectangle, can be drawn around the 1st Stage Separator and all the streams crossing its boundary then feature in balance (or constraint) equations:

$$Ff_c = Gg_c + Ww_c \tag{9}$$

If the left-hand side is replaced with the component flow rather than the product of the wellstream feed total flow and component mole fraction and the equation rearranged:

$$F_c - Ww_c = Gg_c \tag{10}$$

Where,

$$F_c = Ff_c \tag{11}$$

Equation (10) is linear in the unknown variables F_c and W; one such equation can be written for each component.

Once, F_c is determined, F and f_c are simply obtained from:

$$F = \sum_c F_c \tag{12}$$

And,

$$f_c = \frac{F_c}{\sum_c F_c} \tag{13}$$

Also, a total mass balance equation can be written for the control surface in Figure 3:

$$\sum_c F_c - W = G \tag{14}$$

A second control surface is constructed across both separators as depicted in Figure **4**:

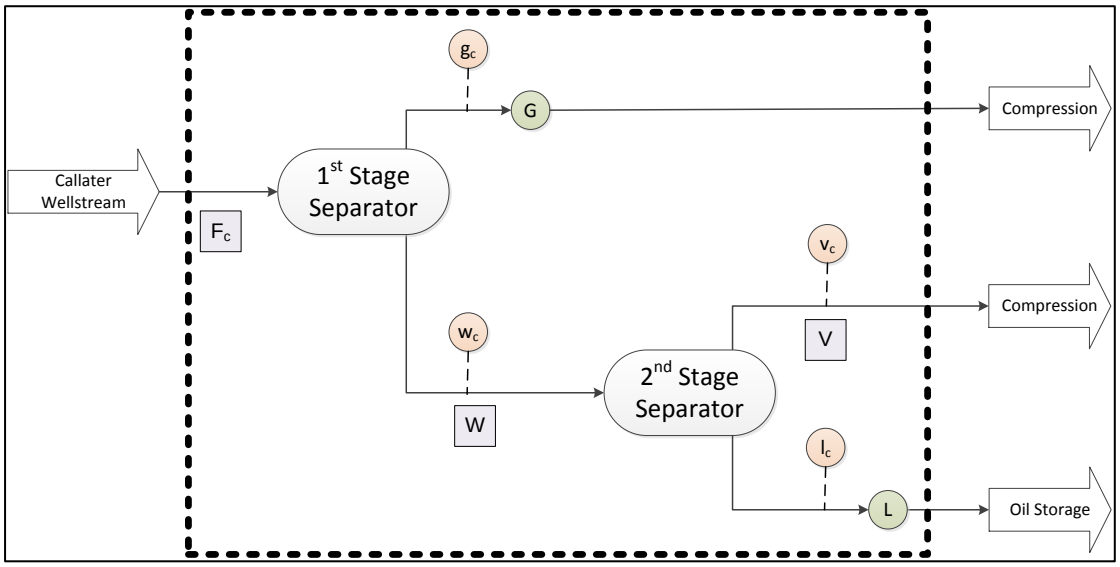

Figure 4 – Process Schematic Process Balance 2

The molar balance across this surface gives:

$$F_c - V v_c = G g_c + L l_c \tag{15}$$

And,

$$\sum_c F_c - V = G + L \tag{16}$$

Which are linear in the unknown variables F_c and V.

Such sets of balances are required so that there are sufficient equations relating the unknown quantity of interest, F_c, to all available measured data.

We now have a system of linear simultaneous equations, which can be written in expanded matrix form as:

$$\begin{pmatrix} 1 & 0 & 0 & -w_1 & 0 \\ 0 & 1 & 0 & -w_2 & 0 \\ 0 & 0 & 1 & -w_3 & 0 \\ 1 & 0 & 0 & 0 & -v_1 \\ 0 & 1 & 0 & 0 & -v_2 \\ 0 & 0 & 1 & 0 & -v_3 \\ 1 & 1 & 1 & -1 & 0 \\ 1 & 1 & 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} F_1 \\ F_2 \\ F_3 \\ W \\ V \end{pmatrix} = \begin{pmatrix} Gg_1 \\ Gg_2 \\ Gg_3 \\ Gg_1 + Ll_1 \\ Gg_2 + Ll_2 \\ Gg_3 + Ll_3 \\ G \\ G + L \end{pmatrix} \tag{17}$$

The above is written for three components but can be expanded to N components.

In compact matrix form this becomes:

$$\boldsymbol{AX} = \boldsymbol{Y} \tag{18}$$

Where,

A is a 2N+2 by N+2 matrix,
X is an N+2 vector and,
Y a 2N+2 vector.

Matrices and vectors (which are matrices with one row or one column) are indicated using emboldened notation.

2N+2 is simply the number of balance equations; for this example, N=3 and the number of balance equations is therefore 8.

The point of formulating the problem in such a matrix format is that the matrix **A** provides information on how many unmeasured variables are observable. This is accomplished by calculating the rank of matrix **A**. The rank is the number of linearly independent rows or columns in the matrix. The maximum value it can be is the lesser of the total number of rows or columns in the matrix. Hence, the maximum for **A** above is 5, which is the number of unknown variables.

With 5 unknowns we only require 5 independent equations to determine their value. In effect, this is a set of 5 simultaneous balance (or constraint) equations in 5 unknowns.

Calculating the rank of a matrix and solving sets of simultaneous linear equations using matrix algebra are tasks readily performed using any matrix software package (which are commonly available).

Hence, from the above set of 8 rows of matrix **A**, we need to choose 5 to form a reduced version of **A**, termed $\mathbf{A_{red}}$, and solve for **X** using:

$$\boldsymbol{X} = \boldsymbol{A_{red}^{-1} Y_{red}} \tag{19}$$

To be invertible $\mathbf{A_{red}}$, has to be a square matrix and hence needs to be 5 x 5. Selecting 5 from the 8 rows, i.e. 5 from the 8 balance equations, may allow the determination of the unknown variables represented by **X** but only if $\mathbf{A_{red}}$ has a rank of 5, otherwise there is not sufficient information and the inverse of $\mathbf{A_{red}}$ is singular.

This alternative formulation can be illustrated by applying it to the simplified example presented in Section 3.2. Table 14 contains the numerical values of matrix **A** (shaded yellow) and vector **Y** (shaded orange) from Equations (17) and (18):

Table 14 – Full Constraint Matrix A and Vector Y

	Matrix A						Vector Y
Constraint	F_1	F_2	F_3	V	W		RHS
A	1	0	0	-0.50	0.00	=	59
B	0	1	0	-0.35	0.00	=	40
C	0	0	1	-0.15	0.00	=	81
D	1	0	0	0.00	-0.09	=	58
E	0	1	0	0.00	-0.23	=	20
F	0	0	1	0.00	-0.68	=	2
G	1	1	1	-1	0	=	180
H	1	1	1	0	-1	=	80

The compositions featuring in the two rightmost columns of **A** are presented as mole fractions, rather than mole %.

The rank of matrix **A** is calculated to be 5, so all 5 unmeasured variables are observable. This calculation informs that there is sufficient data to calculate the unmeasured variables.

The selection of 5 of the 8 constraints, labelled A to H, is now required to produce a square Matrix $\mathbf{A_{red}}$ (and associated $\mathbf{Y_{red}}$).

To illustrate the requirement to have both sets of compositions w_c and v_c, let us select A, B, C, G and H (termed Case 1), thereby omitting all constraints that include w_c:

Table 15 – Reduced Matrix A and Vector Y (Case 1)

	Matrix A_{red}						Vector Y_{red}
Constraint	F_1	F_2	F_3	V	W		RHS
A	1	0	0	-0.50	0.00	=	59
B	0	1	0	-0.35	0.00	=	40
C	0	0	1	-0.15	0.00	=	81
G	1	1	1	-1	0	=	180
H	1	1	1	0	-1	=	80

The matrix $\mathbf{A_{red}}$ has a rank of 4, so is not invertible and there is insufficient information to calculate vector **X**. The reason the rank is 4 is because not all the constraints are independent. By inspection, Constraint G can be formulated by adding A, B and C together, so it provides no additional information and is not independent.

If one constraint is changed, say D is selected instead of C, as shown in Table 16:

Table 16 – Reduced Matrix A and Vector Y (Case 2)

	Matrix A_{red}						Vector Y_{red}
Constraint	F_1	F_2	F_3	V	W		RHS
A	1	0	0	-0.50	0.00	=	59
B	0	1	0	-0.35	0.00	=	40
D	1	0	0	0.00	-0.09	=	58
G	1	1	1	-1	0	=	180
H	1	1	1	0	-1	=	80

Then $\mathbf{A_{red}}$ now has a rank of 5 and **X** is deducible from (19). In fact, so long as at least one constraint involving v_c and one involving w_c are selected then the unknown variables comprising **X** can be determined. In fact, G and H do not need to be selected as illustrated in Case 3:

Table 17 – Reduced Matrix A and Vector Y (Case 3)

	Matrix A_{red}						Vector Y_{red}
Constraint	F_1	F_2	F_3	V	W		RHS
A	1	0	0	-0.50	0.00	=	59
B	0	1	0	-0.35	0.00	=	40
D	1	0	0	0.00	-0.09	=	58
E	0	1	0	0.00	-0.23	=	20
F	0	0	1	0.00	-0.68	=	2

The matrix-based approach above only uses a subset of the component fractions measured across all sampled streams, i.e. just sufficient to solve the equations. The Inferential Method described in Section 3 utilises all the compositional measurements from all sampled streams.

For the perfect simplified examples, both methods calculate exactly the same wellstream feed. With noisy real data they would produce slightly different answers with different uncertainties. Though not calculated in this paper it is anticipated that the original Inferential Method would exhibit a lower uncertainty than the matrix-based approach, since the former maximises the use of the available data.

However, the matrix-based approach could be extended to incorporate all the measurements using data reconciliation techniques as described in [12]. Data reconciliation would also allow the measurement uncertainties to be incorporated to obtain an optimised estimate of the wellstream in terms of its uncertainty.

7 CONCLUSIONS

The accurate determination of the flow and composition of a field's hydrocarbons entering a commingled process is essential to ensure that the field realises the correct value of its allocated production.

In certain circumstances it may be expensive or impractical to install sufficient flow meters to directly measure a field's incoming production. However, the availability of sufficient relevant stream compositional data may afford a method by which the incoming production may be inferred.

This paper described such a method, termed the Inferential Method, to calculate the flow and composition of the Callater field's wellstream which was being tied back to Apache's Beryl Alpha platform. This was carried out, based on an apparently incomplete set of flow measurements, by utilising available compositional data both of streams whose flow was measured and unmeasured.

This was accomplished using only component mass/molar balance equations. The method was extended to incorporate a process simulation to calculate one of the stream compositions which transpired to be difficult to sample in practice.

The Inferential Method calculated wellstream exhibits a greater uncertainty than that associated with the case in which new metering is installed to determine the Callater flow, termed the Direct Measurement Method. The difference in uncertainty was calculated using a Monte Carlo simulation which allowed the increase in the integrated exposure to loss to be calculated in dollar value terms.

This additional exposure to loss was compared to the cost of the new meter in a cost benefit analysis and as such illustrated that the Inferential Method afforded a cost-effective accurate approach to determine Callater's wellstream flow and composition for allocation purposes. The savings to the project, accrued by eliminating the requirement for the new meter, extended into the multiple $100,000's.

The approach was generalised using matrix algebra. This required the conversion of the problem to a linear basis. The advantage of this approach was that it provided a test to determine if unmeasured streams' flows and compositions could be determined from available measurement data and relevant mass balance constraint equations.

The matrix-based approach also exposed the absolute minimum amount of data required and provided an alternative means of performing the calculations. It also provided a framework that could be applied more generally to any process and handle much more complex systems.

In summary: "before investing in new measurement equipment, check if you can infer what you want to measure from data you already have".

8 INFERENTIAL METHOD – MATHEMATICAL DERIVATION

The process schematic Figure 2 is replicated below for ease of reference to the nomenclature.

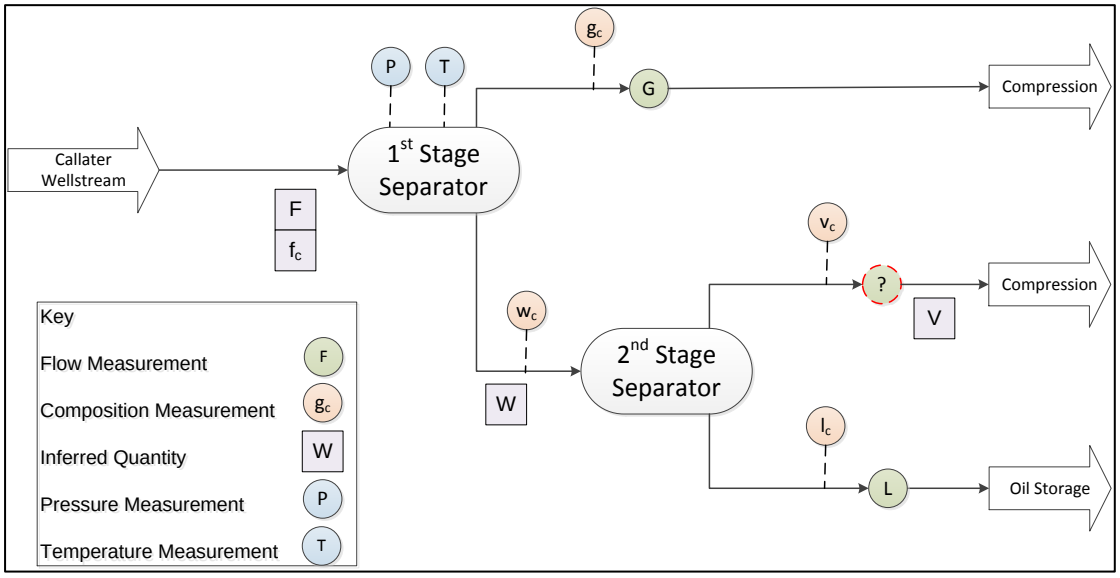

Figure 5 – Beryl Alpha Separator Train Configuration

It has been assumed that all calculations are performed on a dry hydrocarbon basis and that all measurement quantities are net of water. The calculations are performed on a molar basis to allow the method to be extended to incorporate process simulation calculations described in Section 3.3.

A molar balance across the 1st Stage Separator at the total stream level produces:

$$F = G + W \tag{20}$$

And at the component level:

$$Ff_c = Gg_c + Ww_c \tag{21}$$

The quantity β (ratio of first stage vapour to feed rate) is defined as:

$$\beta = \frac{G}{F} \tag{22}$$

Substituting into (20) and rearranging produces:

$$\frac{W}{F} = (1-\beta) \tag{23}$$

Substituting (22) and (23) into (21) and rearranging:

$$f_c = \beta g_c + (1-\beta) w_c \tag{24}$$

This is further rearranged to obtain:

$$f_c = w_c \left(\beta \frac{g_c}{w_c} + (1-\beta) \right) \tag{25}$$

The ratio of g_c to w_c is equivalent to the component K factor used in flash calculations and is expressed in a convenient form here to allow the results of a flash of the 1st Stage Separator hydrocarbons to be incorporated. This rearrangement also allows the compositions of 1st stage vapour and liquid streams to be incorporated with those of the 2nd Stage Separator as described below.

Similar to above, a molar balance across the 2nd Stage Separator at the total stream level produces:

$$W = V + L \tag{26}$$

And at the component level:

$$Ww_c = Vv_c + Ll_c \tag{27}$$

Now define α:

$$\alpha = \frac{V}{W} = \frac{W - L}{W} = 1 - \frac{L}{W} \tag{28}$$

Therefore,

$$\frac{L}{W} = 1 - \alpha \tag{29}$$

Substituting (28) and (29) into (27):

$$w_c = \alpha v_c + (1 - \alpha) l_c \tag{30}$$

Further re-arranging:

$$w_c = l_c \left(\alpha \frac{v_c}{l_c} + (1 - \alpha) \right) \tag{31}$$

This last rearrangement is not strictly necessary but it puts (31) into a similar form to (25) and produces the ratio of v_c to l_c equivalent to the component K factor. Now the w_c outside the brackets in (25) is replaced by the right-hand side of (31) to give:

$$f_c = l_c \left(\alpha \frac{v_c}{l_c} + (1 - \alpha) \right) \left(\beta \frac{g_c}{w_c} + (1 - \beta) \right) \tag{32}$$

Which is rearranged so that the α and β terms appear only once:

$$f_c = l_c \left(1 + \alpha \left(\frac{v_c}{l_c} - 1 \right) \right) \left(1 + \beta \left(\frac{g_c}{w_c} - 1 \right) \right) \tag{33}$$

Noting that β can be expressed as:

$$\beta = \frac{G}{F} = \frac{G}{G+W} \tag{34}$$

Rearranging (29):

$$W = \frac{L}{1-\alpha} \tag{35}$$

And substituting in (34), β can be obtained in terms of α:

$$\beta = \frac{G}{G+\left(\frac{L}{1-\alpha}\right)} = \frac{G(1-\alpha)}{G(1-\alpha)+L} \tag{36}$$

Inserting into (33), allows f_c to be expressed in terms of one unknown, that is α:

$$f_c = l_c\left(1+\alpha\left(\frac{v_c}{l_c}-1\right)\right)\left(1+\left(\frac{G(1-\alpha)}{G(1-\alpha)+L}\right)\left(\frac{g_c}{w_c}-1\right)\right) \tag{37}$$

The sum of the feed composition (f_c) has to equal 1 (or 100%):

$$\sum_c f_c = 1 \tag{38}$$

Therefore:

$$\sum_c l_c\left(1+\alpha\left(\frac{v_c}{l_c}-1\right)\right)\left(1+\left(\frac{G(1-\alpha)}{G(1-\alpha)+L}\right)\left(\frac{g_c}{w_c}-1\right)\right) = 1 \tag{39}$$

Equation (39) can be solved iteratively for α, whose value must lie between 0 and 1. Alternatively α can be determined analytically from:

$$\alpha = \frac{\sum_c \frac{g_c l_c}{w_c} - 1}{\sum_c \frac{g_c l_c}{w_c} - \sum_c \frac{g_c v_c}{w_c}} \tag{40}$$

This is one of the roots of a quadratic equation obtained by rearranging (39).

Once α is determined, f_c is obtained from (37), β from (36), F from (34), W from (35) and V from (28).

If w_c is not measured, then a further outer iteration loop is required. An initial feed composition f_c has to be guessed and using the temperature and pressure in the 1st Stage Separator, the composition of the separator liquid (w_c) is calculated using the flash unit operation representing the separator in the process model. α can then be determined and a new feed composition calculated from (37).

The process iterates until the change in the feed composition is less than a specified tolerance.

If 1st Stage Separator liquid composition has to be estimated from process simulation, then the water content of the 1st stage gas in equilibrium must be accounted for as the water affects the vapour liquid equilibrium in the vessel. Liquid water does not need to be modelled quantitatively as it is effectively immiscible with the liquid oil phase and only affects the hydrocarbon vapour in the water pressure it exerts. Hence, it is assumed that there is sufficient free water present to saturate the vapour.

9 NOTATION

α	Ratio of 2nd stage gas to 1st stage liquid flow
A	Matrix of constraint or balance equation coefficients
$\mathbf{A_{red}}$	Reduced square matrix of constraint or balance equation coefficients
β	Ratio of 1st stage gas to wellstream feed flow
C_1	Component 1, 2, 3 etc., represented similarly
E	Integrated exposure to loss
F	Wellstream feed total flow
F_c	Wellstream feed flow of component c
f_c	Wellstream feed fraction of component c
G	1st stage gas total flow
g_c	1st stage gas fraction of component c
L	2nd stage liquid (or oil) total flow
l_c	2nd stage oil fraction of component c
N	Number of components
P	Pressure
T	Temperature
$U	Absolute uncertainty (in wellstream dollar value)
V	2nd stage gas total flow
v_c	2nd stage gas fraction of component c
W	1st stage liquid (or oil) total flow
w_c	1st stage oil fraction of component c
X	Vector of unmeasured variables
Y	Vector of constraint or balance equation terms (RHS)
$\mathbf{Y_{red}}$	Vector of reduced set of constraint or balance equation terms (RHS)

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